Various ways of introducing herding behavior into the agent based models of complex systems

Petras Purlys[1,2], Aleksejus Kononovicius[2] and Vygintas Gontis[2]

[1] - Faculty of Physics, Vilnius University

[2] - Institute of Theoretical Physics and Astronomy, Vilnius University

petras.purlys@gmail.com

Abstract

Herding behavior in the complex systems is a very interesting and important topic. This claim is backed by the essential property of such systems - the interactions between distinct parts of the complex system is of the utmost importance to understand its macroscopic behavior. Generally speaking there may be two very distinct ways of defining herding behavior - emphasis on the global coupling and emphasis on the individual interactions. In this contribution we will study these two different approaches and show the similarity of the asymptotic behavior of the approaches.

Considered models

In this contribution we will considered Kirman's model, which emphasizes the interaction between two individual agents, [1] and the GLM model, which considers the interaction with whole community or its certain, yet rather large, parts, [2]. The ideological scheme of the difference in agent interaction in the aforementioned models is given in Fig. 1.

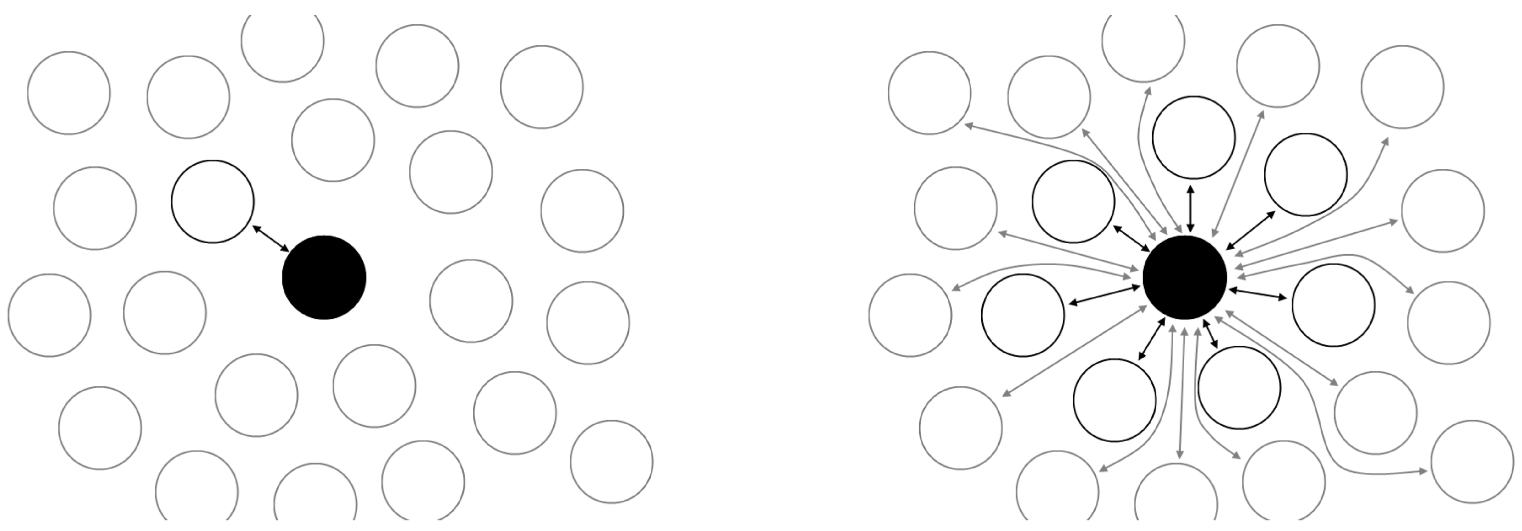

Figure 1: Interactions occurring in the Kirman's (left) and GLM (right) models per single time step, Δt.

Derivation of Kirman's herding model was inspired by the empirical research done by the entomologists (see recent review in [3]). In Fig. 2 we show one of the possible experimental setups, two identical paths connecting ant colony and food source, and ideological scheme behind the Kirman's model.

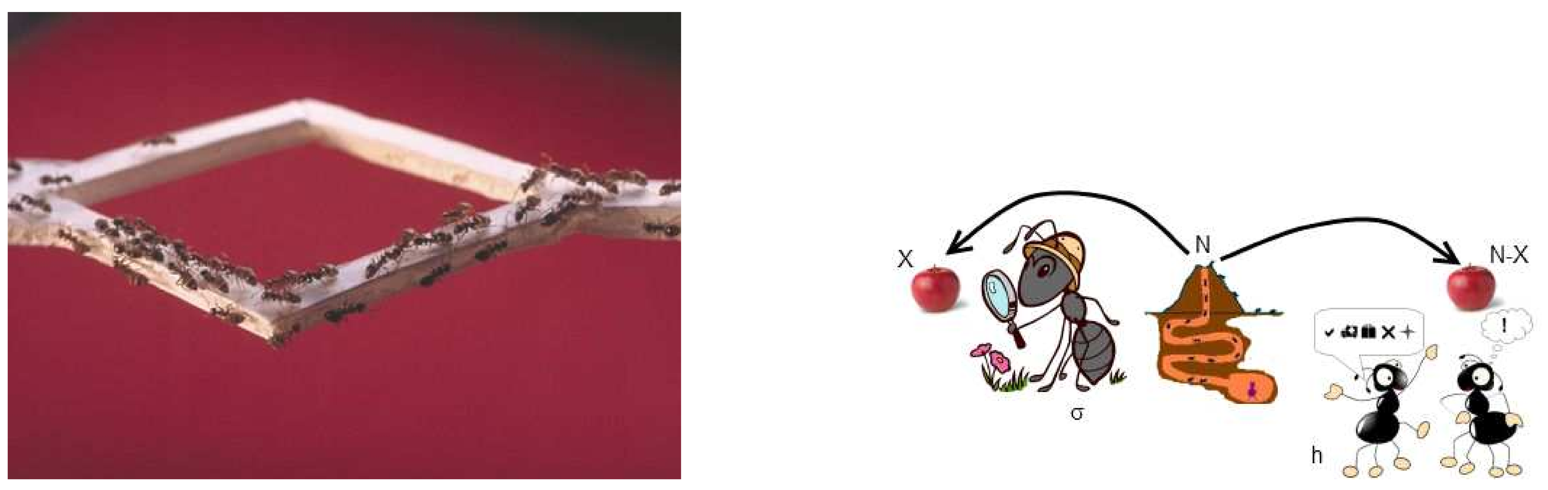

Figure 2: Empirical (left; taken from [3]) and model (right) setup of herding in the ant colonies.

Mathematically Kirman's ideas presented on the right sub-figure are formulated as one step transition probabilties [4]:

$$p(X \to X+1) = (N-X)(\sigma_1 + hX)\Delta t, \quad p(X \to X-1) = X[\sigma_2 + h(N-X)]\Delta t, \tag{1}$$

where X is a number of ants using the chosen path, the purpose of σ and h terms should be evident from the sub-figure.

Alternative approach was proposed by Goldenberg group [2]. They considered production diffusion scenario and expressed per-agent product adoption probability as:

$$p_{GLM}(s_1 \to s_2) = 1 - (1-\Sigma)(1-H)^{N-X}, \tag{2}$$

which reads as probability of the cumulative event opposite to the not adopting due to individual prefference, $1-\Sigma$, or not being recruited by the agents who have already adopted, $(1-H)^{N-X}$. Putting it more simply - its a probability that at least one agent recruited the considered agent. Note that the adoption probability is defined in certain time window, Δ, thus H and Σ are linear functions of Δt [5].

Asymptotic behavior of the Kirman's and GLM models

Asymptotic behavior of the unidirectionally formulated Kirman's and GLM models is known [5, 6]. In the limit of small time steps they both converge towards the Bass diffusion model:

$$\partial_t X(t) = (N-X)\left[\sigma + \frac{h}{N}X(t)\right], \quad X(0) = 0. \tag{3}$$

While for larger Δt Kirman's model becomes undefined, while GLM model starts to behave as lagged Bass model:

$$\partial_t X(t) = (N-X)\left[\sigma + \frac{h}{N}X(t-\theta)\right], \quad X(t) = 0,\ t < \theta, \tag{4}$$

where $\theta = \Delta t/2$ should hold. The obtained agreement is shown in Fig. 5.

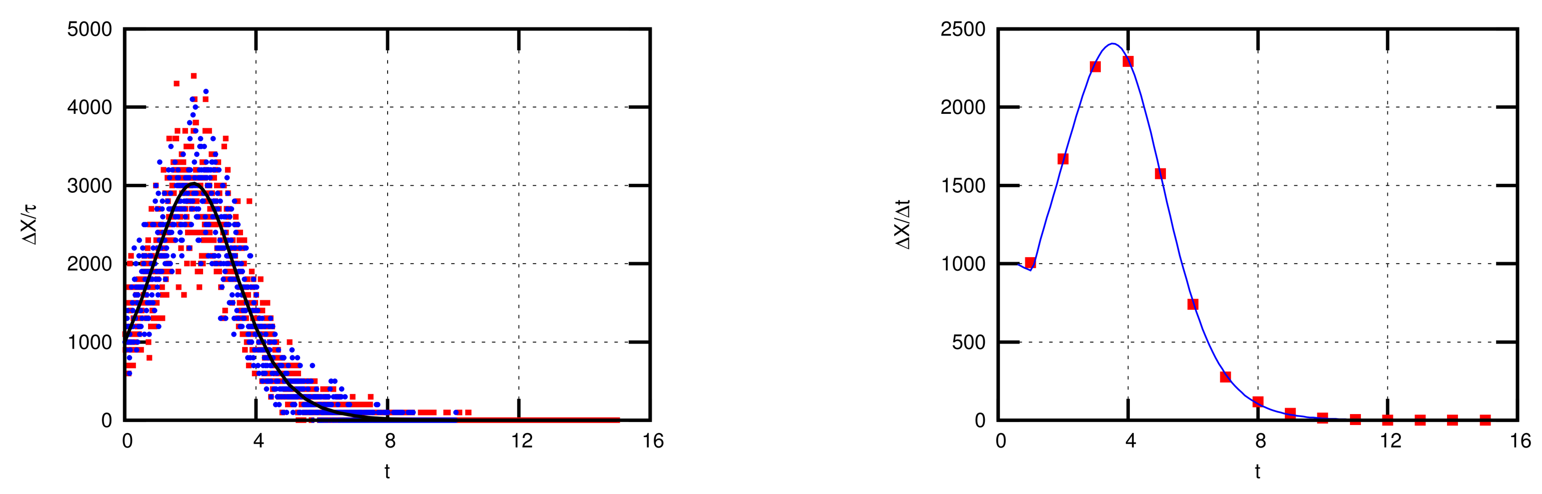

Figure 3: Kirman's (blue dots), GLM (red dots) model results fitted with the original Bass model (black curve) in the limit of small Δt (left). GLM (red dots) model results fitted by the lagged Bass (blue curve) model for the larger Δt (right). Model parameters were set as follows: $\sigma = 0.1$, $h = 1$, $N = 10^4$ (in all cases), $\tau = 0.01$ and $\Delta t = 10^{-8}$ (left), $\tau = \Delta t = 1$ (right).

In case of the bidirectional transition Kirman's model is well approximated by the following stochastic differential equation [4, 7]:

$$\mathrm{d}x = [\sigma_1(1-x) - \sigma_2 x]\,\mathrm{d}t + \sqrt{2hx(1-x)}\mathrm{d}W, \tag{5}$$

where $x = X/N$ and W is a standard Brownian motion.

The bidirectional GLM model formulation is not known, though it can be easily obtain by relying on the Kirman's herding ideas yielding following per-agent transition probabilities:

$$p(s_1 \to s_2) = 1 - (1-\sigma_1\Delta t)(1-h\Delta t)^{N-X}, \quad p(s_2 \to s_1) = 1 - (1-\sigma_2\Delta t)(1-h\Delta t)^{X}. \tag{6}$$

By expanding them in the limit of small Δt one obtains

$$p(s_1 \to s_2) \approx [\sigma_1 + h(N-X)]\Delta t, \quad p(s_2 \to s_1) \approx [\sigma_2 + hX]\Delta t. \tag{7}$$

One can go even further by assuming that Δt is actually small enough for only single transition to be probable. In such case one can use the averages of the Binomial distribution to obtain one step transition probabilities for the GLM model, which in fact coincide with (1). Consequently

$$\Delta X = \sum_{i=1}^{N-X} \psi_i[p(s_2 \to s_1)] - \sum_{i=1}^{X} \psi_i[p(s_1 \to s_2)], \tag{8}$$

can be shown to be well approximated by the (5) in the same limit. Note that the approximation of the GLM model by (5) is sufficient only if $\Delta t < N^{-2}$. For larger time steps herding tendencies in the GLM model become weaker or fully disappear (see Fig. 4).

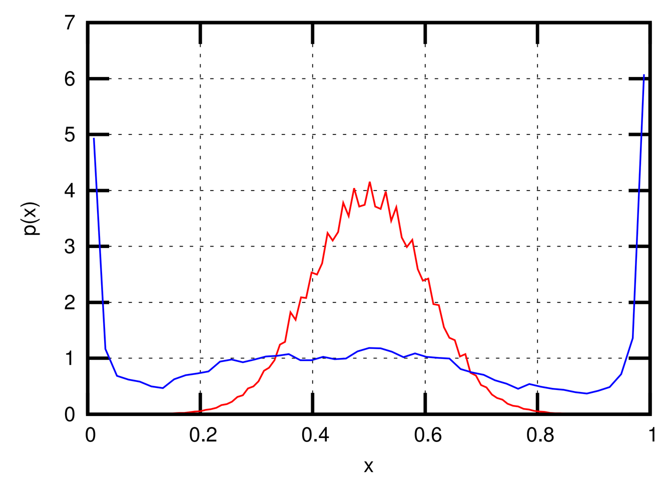

Figure 4: Stationary probability density function of population fraction, x, in case of strong herding behavior then the model is solved using small, $\sim N^{-2}$, (blue curve) and large, $\sim N^{-1}$, (red curve) Δt values. Model parameters were set as follows: $\sigma_1 = \sigma_2 = 0.1$, $h = 1$, $N = 10^3$, $\kappa = 0.1$, $\tau_{disc} = 10^{-4}$.

Further we can move towards the GLM model for the absolute return in financial markets. Wallrasian scenario under the assumption of instantaneous clearing suggests that the absolute return, y, [4] and its statistical features are given by [7]:

$$y = \frac{x}{1-x}, \quad p(y) \sim y^{-\frac{\sigma_2}{h}-1}, \quad S_y(f) \sim f^{-\frac{\sigma_2}{h}+1}. \tag{9}$$

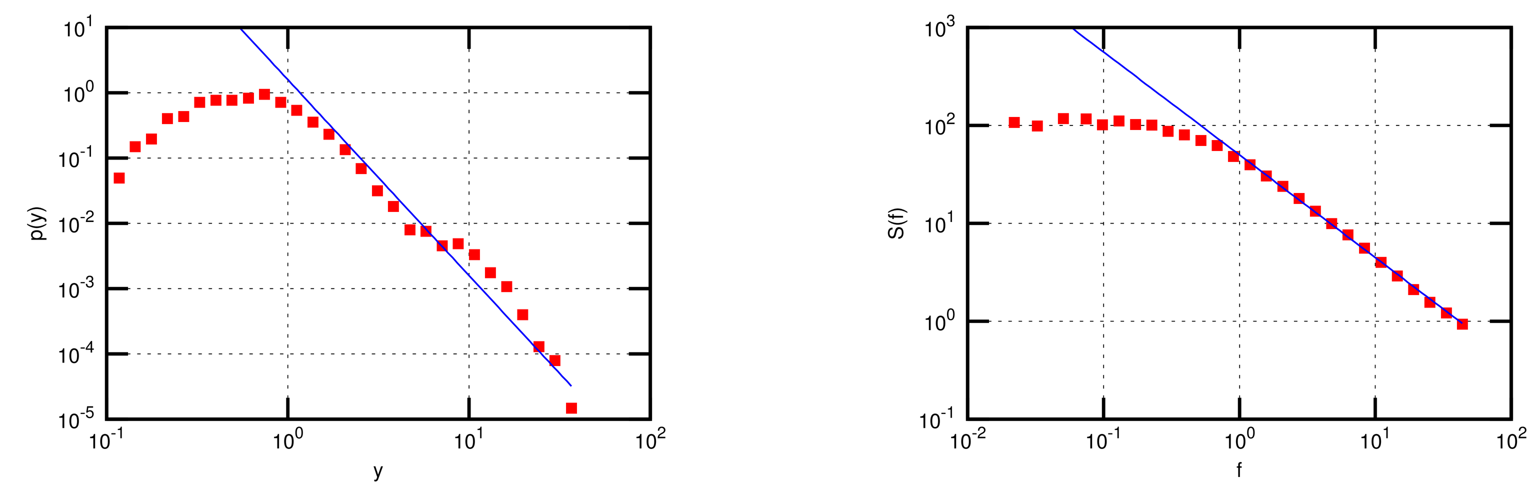

Figure 5: Statistical features probability density function (left) and spectral density (right) of y (red dots) fitted by the theoretical predictions obtained in [7] (blue curves). Model parameters were set as follows: $\sigma_1 = \sigma_2 = 2$, $h = 1$, $N = 10^3$, $\Delta t = 10^{-7}$, $\tau_{disc} = 10^{-2}$.

Conclusions

- GLM and Kirman's models, in both unidirectional and bidirectional formulations, are equivalent in the limit of small Δt. In the same limit both models agree with Bass diffusion model.
- For the larger Δt values the original GLM model might be reconciled with the Bass diffusion model by introducing time lag into the herding behavior. While the bidirectional GLM model does not posses correct scaling properties and thus only works well in the same limit Kirman's model does.
- The bonding between GLM and Kirman's models serves as a proof that polling whole population and random individuals is statistically equivalent.

References

[1] A. P. Kirman, Ants, rationality and recruitment, Quarterly Journal of Economics **108**, 137-156 (1993).

[2] J. Goldenberg, B. Libai, E. Muller, Using Complex Systems Analysis to Advance Marketing Theory Development, Academy of Marketing Science Review **01** (9), 2001.

[3] C. Detrain, J. L. Deneubourg, Self-organized structures in a superorganism: do ants behave like molecules?, Physics of Life Reviews **3**, 162-187 (2006).

[4] S. Alfarano, T. Lux, F. Wagner, Estimation of Agent-Based Models: The Case of an Asymmetric Herding Model, Computational Economics **26** (1), 19-49 (2005).

[5] G. Fibich, R. Gibori, E. Muller, *A Comparison of Stochastic Cellular Automata Diffusion with the Bass Diffusion Model* (NYU Stern School of Business, 2010).

[6] V. Daniunas, V. Gontis, A. Kononovicius, Agent-based versus macroscopic modeling of competition and business processes in economics, ICCGI 2011, The Sixth International Multi-Conference on Computing in the Global Information Technology, 84-88 (2011).

[7] A. Kononovicius, V. Gontis, Agent based reasoning for the non-linear stochastic models of long-range memory, Physica A **391**, 1309-1314 (2012).