

# Kuo fizikams įdomios socialinės sistemos?

## Aleksejus Kononovičius

Vilniaus universitetas Fizikos fakultetas Teorinės fizikos ir astronomijos institutas

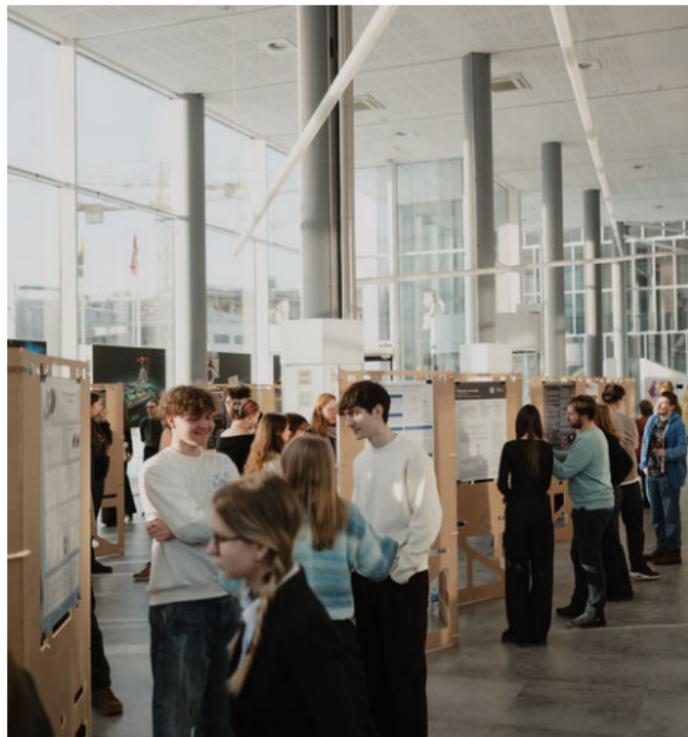
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**Fizikos  
fakultetas**

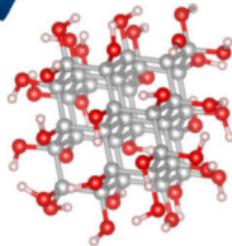
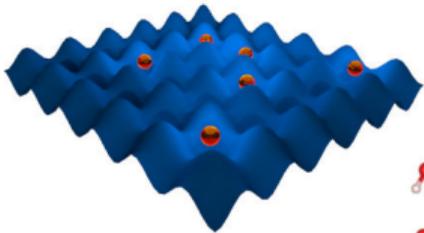
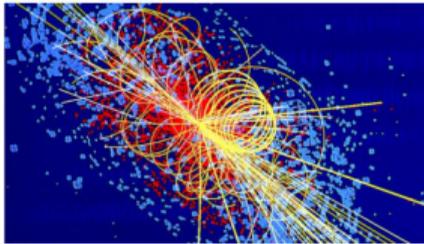




- Seniausio rytų Europos universiteto fakultetas.
- Visų pakopų studijų programos.
- Nuo teorinės fizikos iki medžiagų mokslo ir technologijų vadybos.
- Virš pusės šimto laboratorijų.
- Aktyvus studentų gyvenimas: OpenReadings, FiDi, energetikų ir astronomų klubai.
- Moksleiviams: “Studentas vienai dienai”, “Fotono” dienos stovykla.

# Teorinės fizikos ir astronomijos institutas

Nuo labai mažų ( $10^{-18} - 10^{-8}$  m) iki labai didelių ( $10^8 - 10^{21}$  m)



# Kompleksinių fizinių ir socialinių sistemų grupė

## Socialinės sistemos

Žmonės yra:

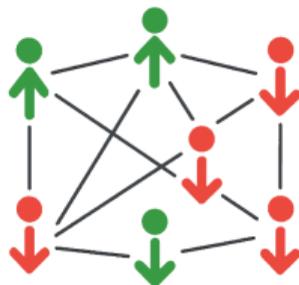
- Išradingi
- Savanaudžiai
- Racionalūs



Tikrai taip ir niekaip kitaip.

Atomai:

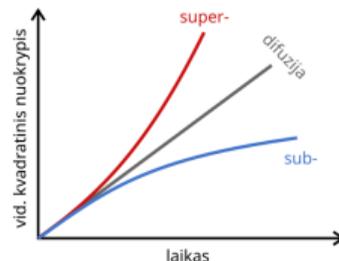
- fantazijos trūkumas
- universalūs dėsniai



Netokie jau mes kitokie...

Kitos temos:

- Ilga atmintis
- Anomali difuzija
- Kvantinių grupių įvaidžių teorija
- Netiesinės sistemos



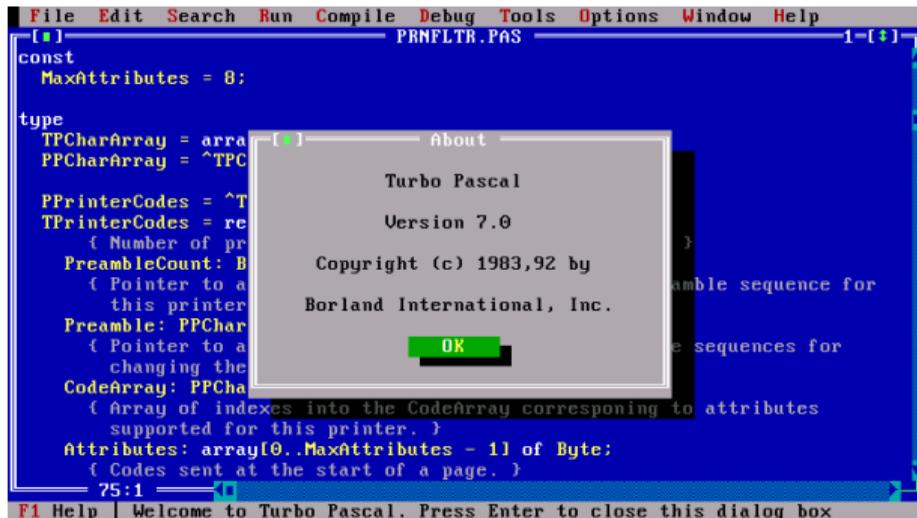
KAL's cartoons, ChatGPT įkvėptas "spinson" minties (K. Sznajd-Weron group)



Mano kelias į fiziką



# Programavimas



The screenshot shows the Turbo Pascal IDE with a code editor window titled 'PRNFLTR.PAS'. The code defines constants and types for printer attributes. An 'About' dialog box is overlaid on the code, displaying the following text:

```
File Edit Search Run Compile Debug Tools Options Window Help
PRNFLTR.PAS
const
  MaxAttributes = 8;

type
  TPCharArray = array[0..MaxAttributes-1] of Byte;
  PPCharArray = ^TPCharArray;

  PPrinterCodes = array[0..MaxAttributes-1] of Byte;
  TPrinterCodes = record
    PreambleCount: Byte;
    Preamble: PPCharArray;
    CodeArray: PPCharArray;
    Attributes: array[0..MaxAttributes-1] of Byte;
  end;

  ( Number of printer codes supported for this printer. )
  ( Pointer to a printer code sequence for this printer. )
  ( Pointer to a printer code sequence for this printer. )
  ( Array of indexes into the CodeArray corresponding to attributes supported for this printer. )
  ( Codes sent at the start of a page. )

75:1
```

The 'About' dialog box contains the following text:

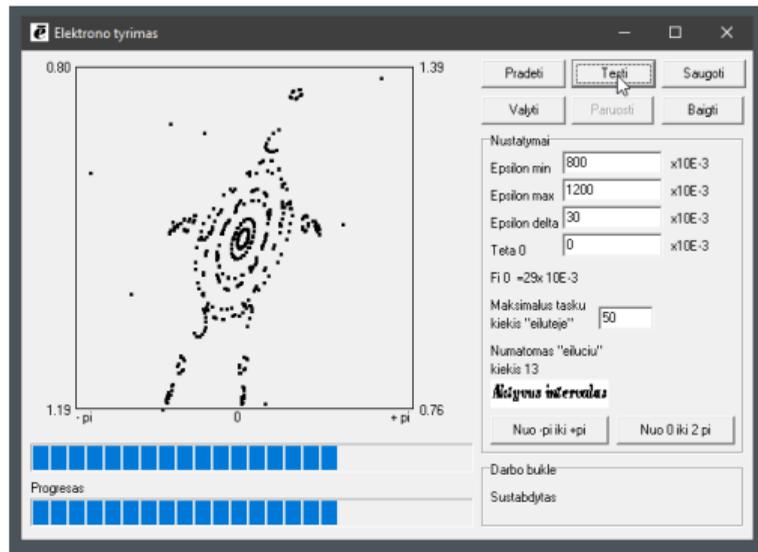
About  
Turbo Pascal  
Version 7.0  
Copyright (c) 1983,92 by  
Borland International, Inc.  
OK

At the bottom of the IDE, a status bar reads: 'F1 Help | Welcome to Turbo Pascal. Press Enter to close this dialog box'.



 Borland Int., Borland Int.

# Atsitiktinumas



📧 (koliažas) Saulius Kazlauskas + Lietuvos jaunųjų mokslininkų sąjunga

Physics of Risk About Students

Search using Google

## Boundary conditions and zealotry in the noisy voter model

January 13, 2026 Aleksejus Kononovicius #interactive #agent-based models #opinion dynamics #voter model  
#boundary conditions #zealots #birth-death process

When writing our recent article [1], we (I and Rytis) had argued about the relationship between having stable voter base (zealots) and imposing boundary conditions on the noisy voter model. This post explores the subtle differences between these notions.

### The noisy voter model

Let us assume that we have  $N$  agents. Each of the agents can be in state 0 or in state 1. In general, the labeling of the chosen such labels to emphasize that we observe the number of agents in state 1, let



s, it will either increase or decrease by a single unit. I.e., one agent will either move all this transition "death", or the agent will move  $0 \rightarrow 1$  (let us call this transition strange names for the rates to emphasize that we can use continuous time birth-death

either independently (with rate  $\varepsilon_0$  or  $\varepsilon_1$ ), or due to peer interaction (let the interaction e birth rate of the model is given by,

$$\lambda(X) = (N - X)(\varepsilon_1 + X). \quad (1)$$


Risk Physics

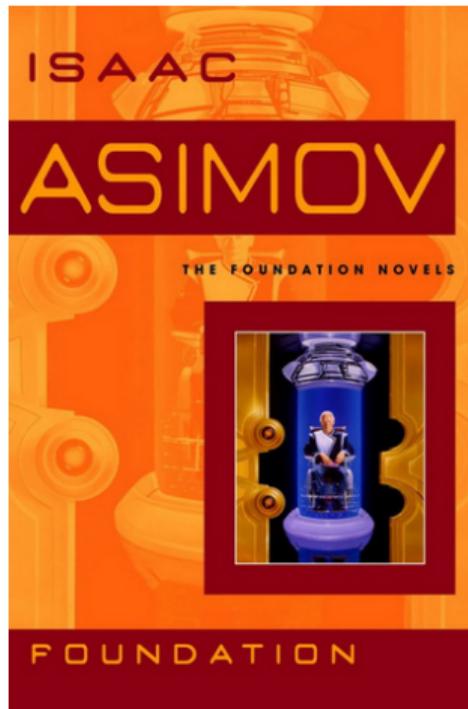
Physics of risk, complexity and socio-economic systems.

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- Theme @ GitHub

- Vilnius University
- Faculty of Physics
- Institute of Theoretical Physics and Astronomy
- COST P10 meeting in Vilnius (2006)
- DPG Physics of Socio-

# Planas

- 1 Mano kelias į fiziką
- 2 Mastelio klausimas
- 3 Laipsniniai skirstiniai
- 4 Rinktiniai modeliai

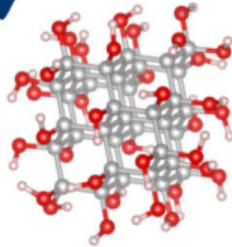
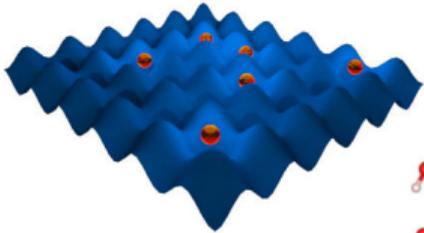
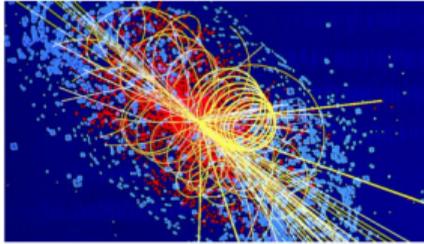


 Bantam Books, Apple TV+

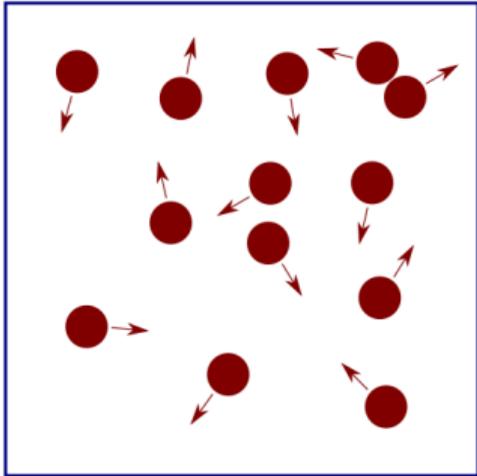
A background network diagram consisting of numerous white circular nodes connected by thin, light blue lines. The nodes are arranged in a complex, interconnected pattern across the entire page. A dark blue horizontal bar is positioned in the center, containing the text "Mastelio klausimas".

# Mastelio klausimas

# “Paprastos” sistemas



# Sudėtingos sistemos

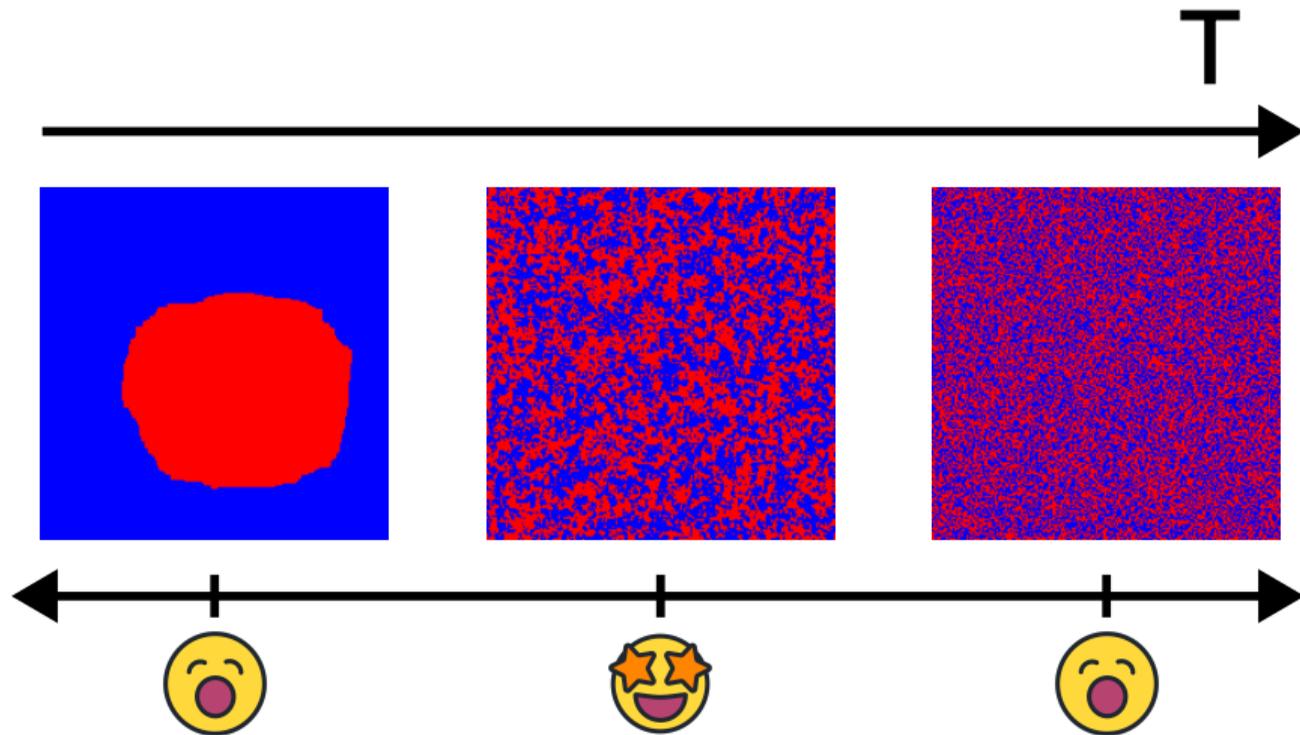


“Visuma yra šis tas daugiau nei atskirų dalių suma” (Kurtas Kofka\*)

 Robina Weermeijer, KAL's cartoons

\* nors dažnai sakoma, kad Aristotelis ar Euklidas

# Kritinio taško keistenybės



# Kaip suvokti “mastelį”?

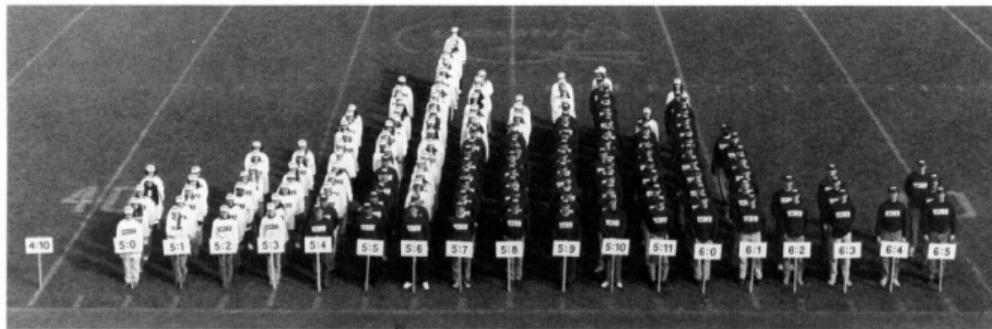
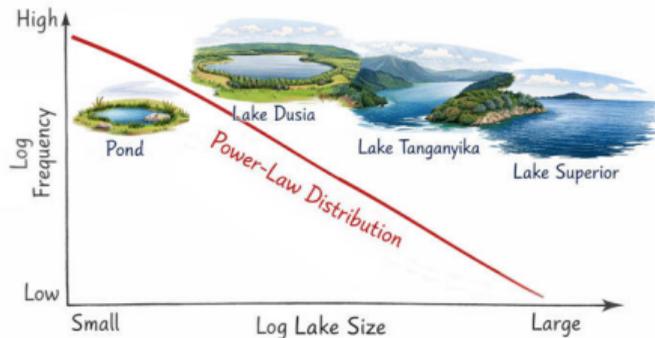


Figure 7. Living histogram of 143 student heights at University of Connecticut.



Kokio ūgio yra  
“tipinis” žmogus?

Kokio ploto yra  
“tipinis” ežeras?

# Pakrantės ilgio paradoksas

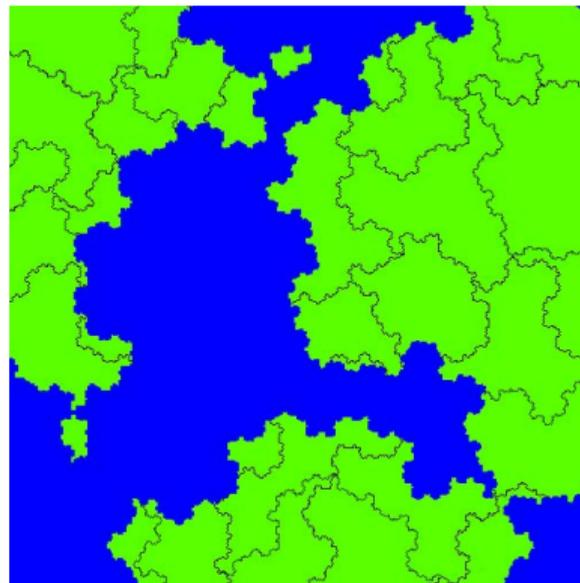
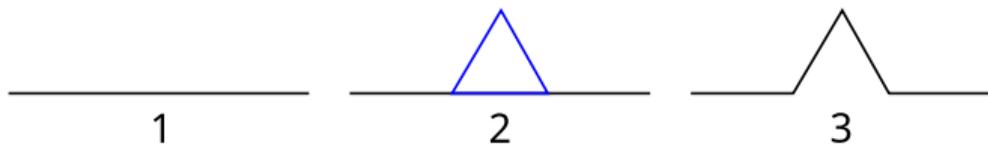


- Liniuotė – 200 km.  
Ilgis – 2400 km.
- Liniuotė – 100 km.  
Ilgis – 2800 km.
- Liniuotė – 50 km.  
Ilgis – 3450 km.

# Kocho snaigė

Kiekviename žingsnyje kiekvienai tiesei:

- 1 Padalijame į tris dalis.
- 2 Įterpiame lygiakraštį trikampį.
- 3 Pašaliname trikampio pagrindą.



Mano senas bandymas kurti  
kompiuterinį žaidimą.

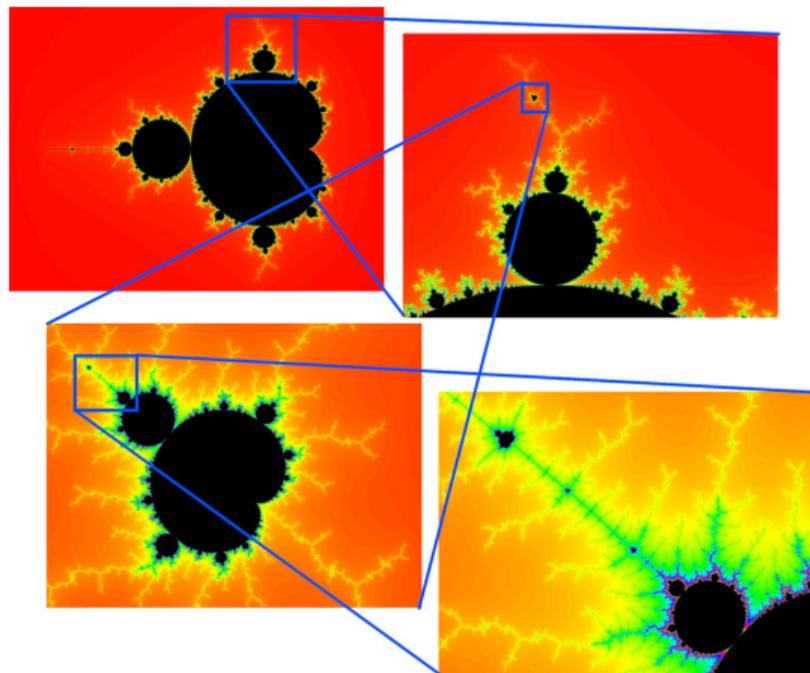
# Ką reiškia neturėti mastelio?

**Matematiškai** funkcija  $f(x)$  neturi mastelio, jei kiekvienam  $b > 0$  ir  $x > 0$  galioja

$$f(bx) = g(b) \cdot f(x).$$

Kitaip tariant, artinant ar tolinant

- $f(x)$  forma nekinta,
- kinta tik amplitudė.



Mandelbroto aibės fragmentai

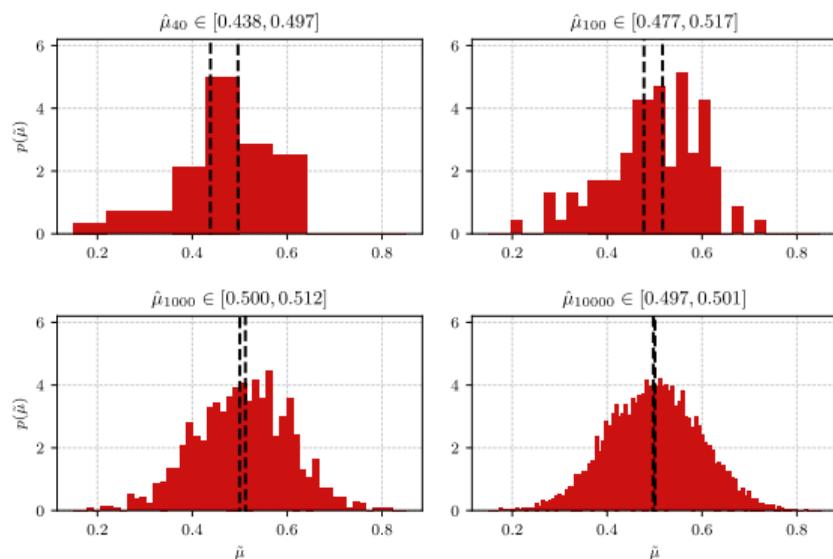
A background network diagram consisting of numerous white circular nodes connected by thin, light blue lines. The nodes are arranged in a complex, interconnected pattern across the entire page.

# Laipsniniai skirstiniai

# Kam fizikams statistika?



# Daugiau matavimų – tikslesnis rezultatas



Stulpeliai – atliktų matavimų tankio histograma.  
Punkturu – įverčio pasikliauties intervalo ribos.

Įvertis:

$$\tilde{\mu}_N = \frac{\sum_{i=1}^N \mu_i}{N}.$$

Matematinė viltis:

$$\mathbb{E}[\tilde{\mu}_N] = \frac{\sum_{i=1}^N \mathbb{E}[\mu_i]}{N} = \mathbb{E}[\mu].$$

Įverčio dispersija\*:

$$\mathbf{Var}[\tilde{\mu}_N] = \frac{1}{N^2} \sum_{i=1}^N \mathbf{Var}[\mu_i] = \frac{\mathbf{Var}[\mu]}{N}.$$

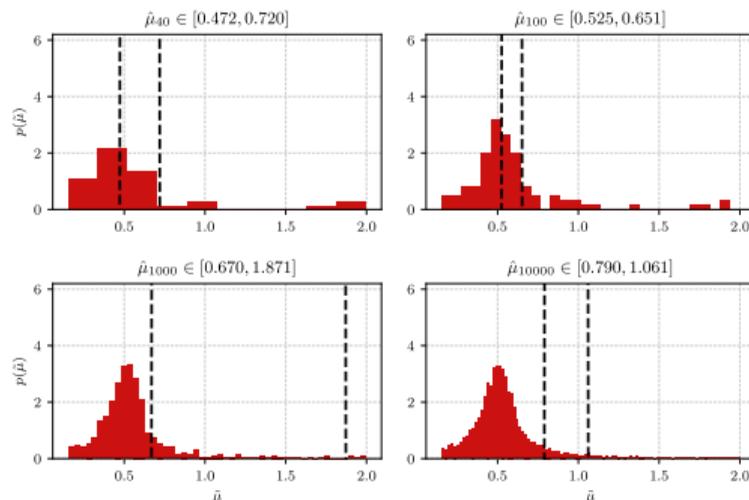
\* – išraiška galioja tik nepriklausomiems eksperimentams.

# O jei nėra mastelio\*?

Paprastiausiu atveju ( $x_0 = 0$ ,  $\gamma = 1$ ),  
**Košį** (angl. *Cauchy*) **skirstinio**  
tikimybės tankio funkcija,

$$p(x) = \frac{1}{\pi(1+x^2)}.$$

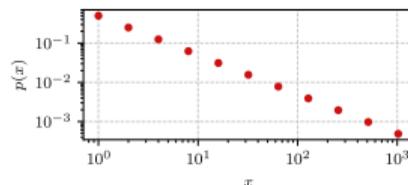
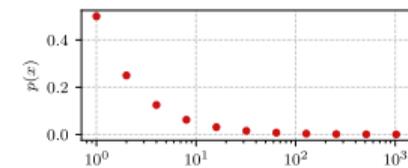
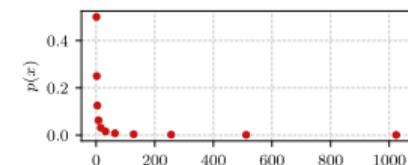
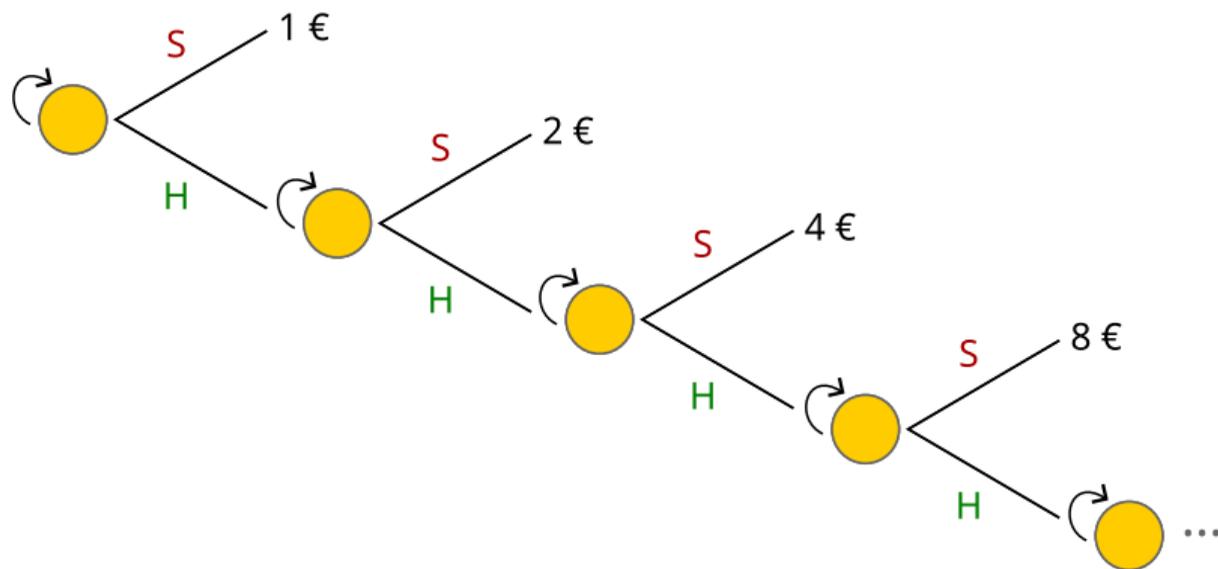
Šio skirstinio statistiniai momentai  
yra neapibrėžti.



Stulpeliai – atliktų matavimų tankio histograma.  
Punktyru – įverčio pasikliauties intervalo ribos.

\* – Koši skirstinys turi parametrą vadinamą masteliu, bet neturi dispersijos.

# Sankt Peterburgo paradoksas



Kiek tikėtumėtės laimėti tokiame žaidime? Kaip pavaizduotumėte tikimybės laimėti tam tikras sumas?

# Pareto skirstinys

Pareto skirstinio, vertėms  $x \in [1, \infty)$ , tikimybės tankio funkcija

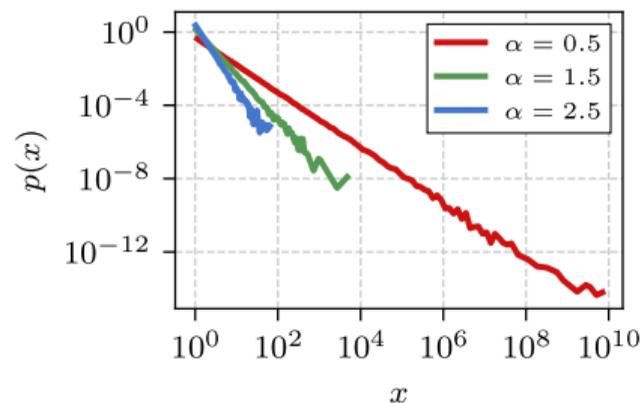
$$p(x) = \alpha x^{-\alpha-1}.$$

Koši skirstinys turi uodegas panašias į Pareto skirstinį su  $\alpha = 1$ .

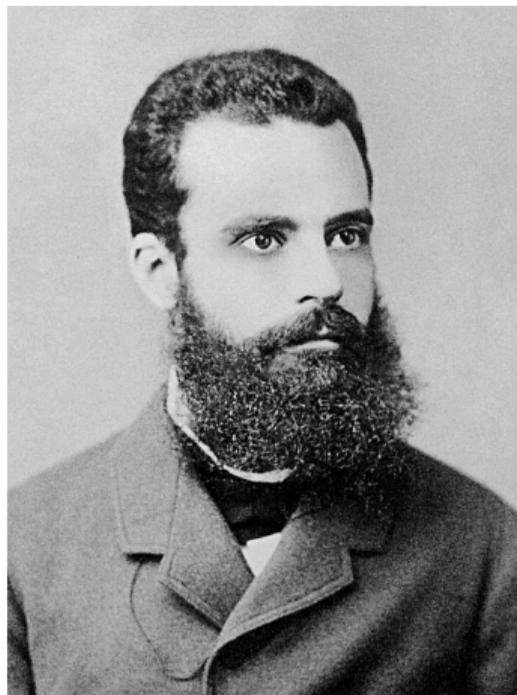
Pareto skirstinio  $k$ -tasis momentas,

$$\mathbb{E} [x^k] = \int_0^{\infty} x^k p(x) dx = \alpha \int_1^{\infty} x^{k-\alpha-1} dx = \frac{\alpha}{k-\alpha} x^{k-\alpha} \Big|_1^{\infty}.$$

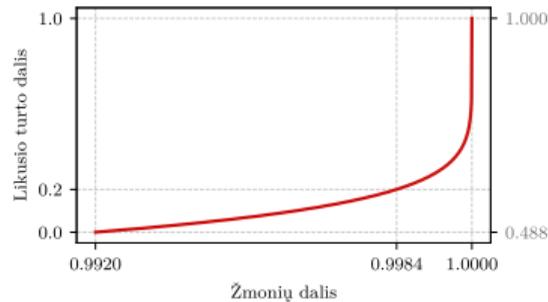
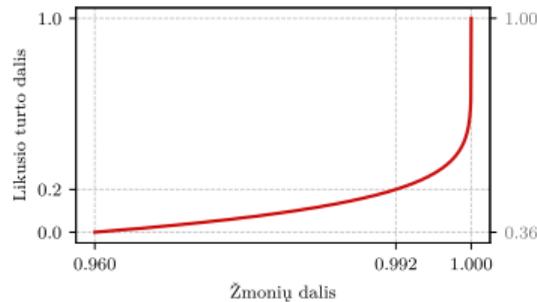
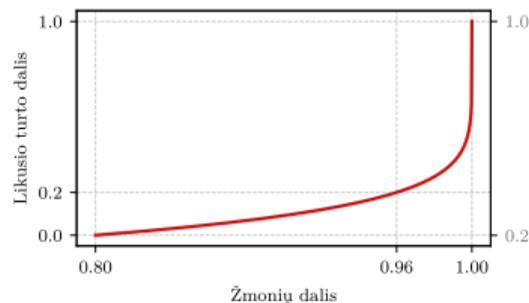
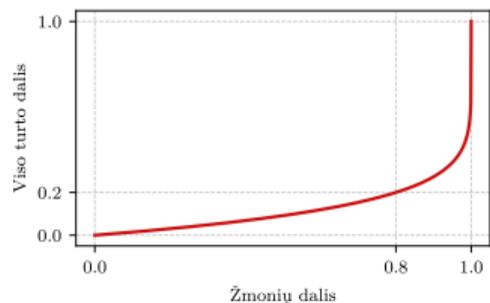
Kad  $k$ -tasis momentas būtų baigtinis, turime reikalauti, kad  $\alpha > k$ .



# Pareto (80–20) principas neturi mastelio

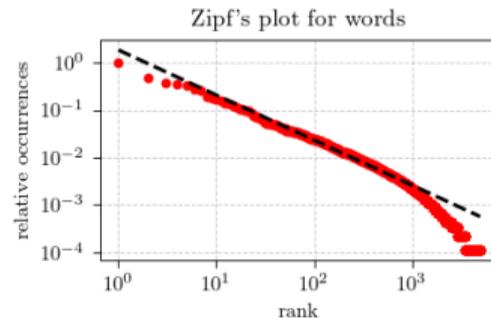
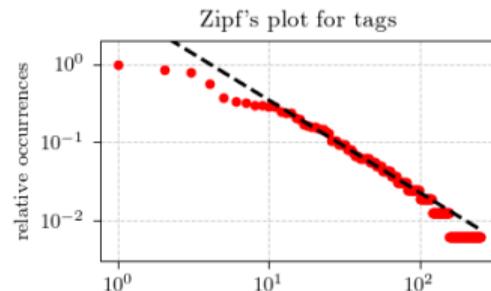
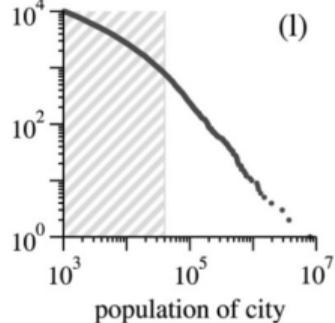
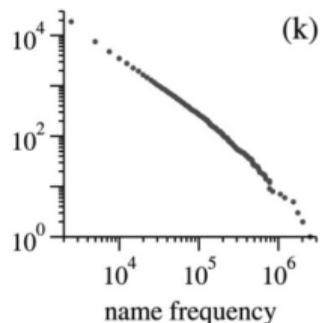
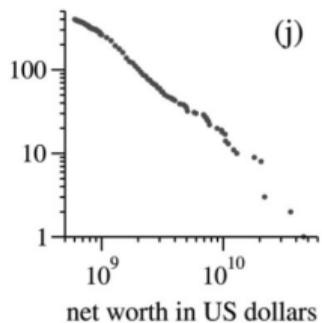
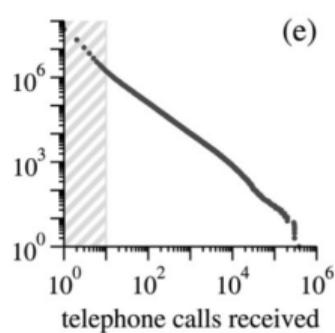
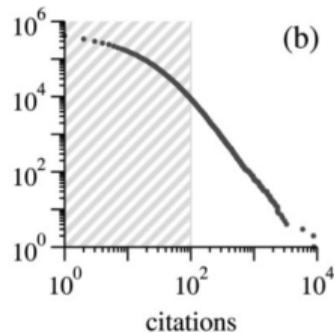
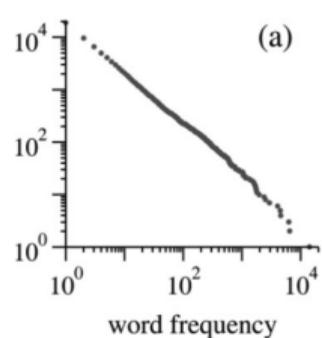


Vilfredo Pareto (1848–1923)



Principo atkūrimas ( $\alpha = 1.16$ ): 1% turtingiausių valdo 50% turto

# Laipsniniai skirstiniai yra visur



A background network diagram consisting of numerous white circular nodes connected by thin, light blue lines. The nodes are arranged in a complex, interconnected pattern across the entire page. A dark blue horizontal bar is positioned in the center, containing the text 'Rinktiniai modeliai' in white.

# Rinktiniai modeliai

# Stebimo dydžio transformacijos

Tegu:

- $y$  nestebimas ir yra pasiskirstęs

$$p_y(y) = \lambda \exp(-\lambda y).$$

- $x$  yra stebimas ir priklauso nuo  $y$ ,

$$x = \exp(y).$$

Tuomet,  $x$  bus pasiskirstęs

$$p_x(x) = p_y(\ln(x)) \left| \frac{dy}{dx} \right| = \lambda x^{-1-\lambda}.$$

```
[1]: import numpy as np
import matplotlib.pyplot as plt

from stats.pdf import make_pdf, make_log_pdf

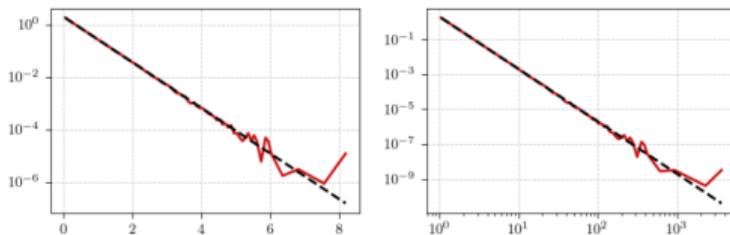
[2]: np.random.seed(5258)

la = 2.0
n_points = 1000000

y = np.random.exponential(scale=1/la, size=n_points)
x = np.exp(y)

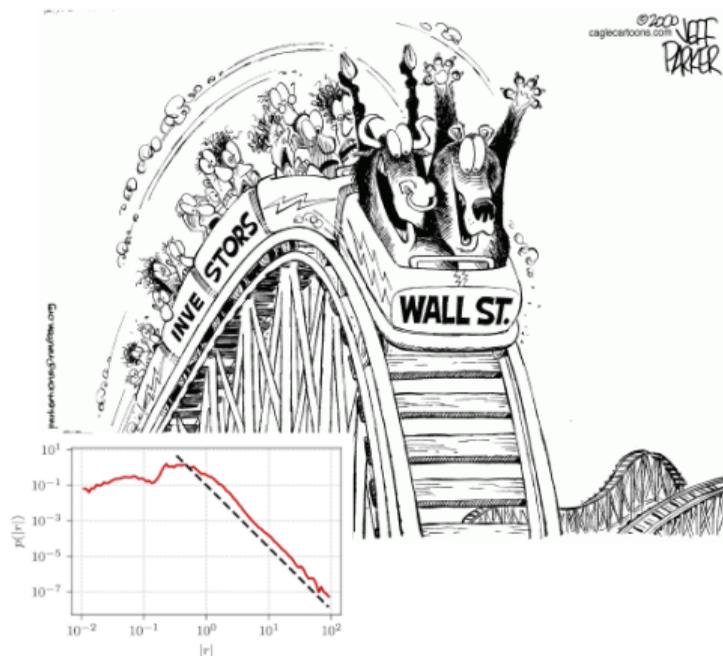
pdf_y = make_pdf(y) # mano funkcija
pdf_x = make_log_pdf(x) # mano funkcija

[3]: ...
```



☰ [Newman (2005)]; 🗣️ Apie eksponentinį (Physics of Risk)

# Finansiniai svyravimai = ...



Tegu yra du prekiautojų tipai:

- Dalis prekiauja pagal savo nuotaiką:

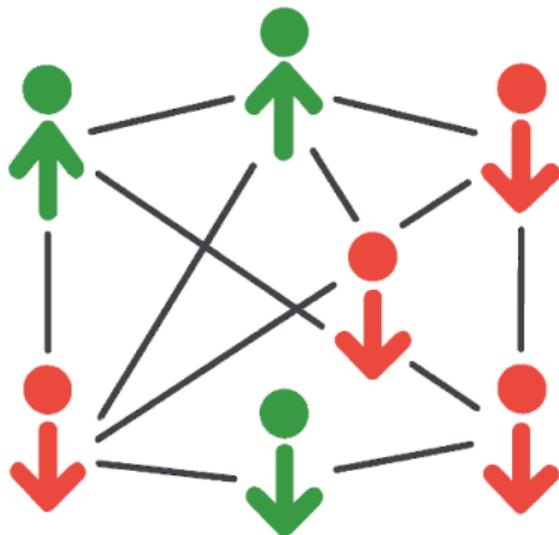
$$D_c(t) = -r_0 X_c(t) \xi(t).$$

- Dalis pagal tikrąją vertę,  $P_f$ :

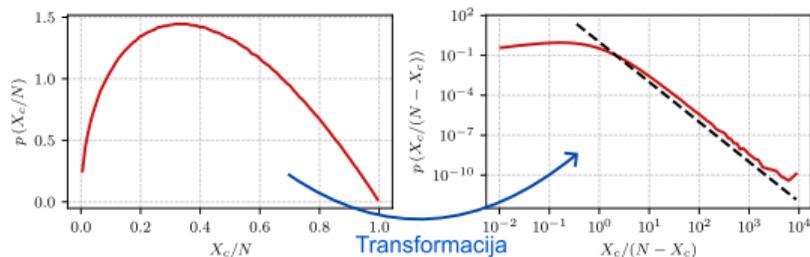
$$D_f(t) = X_f(t) \ln \left[ \frac{P_f}{P(t)} \right].$$

Nesunku parodyti, kad log-pokyčiai finansų rinkose,  $|r| \propto X_c/X_f$ .

# ... = nuomonių svyravimai



- Prekeiviai gali būti “c” ir “f” būsenose.
- Prekeiviai gali patys keisti būseną.
- Prekeiviai gali kopijuoti kitus prekeivius.



📺 ChatGPT įkvėptas “spinson” minties (K. Sznajd-Weron group)

📖 [Kononovicius and Gontis (2012)]; 🗣️ Lithuanian elections, Financial markets (Physics of Risk)

# “Persiklojantys” procesai

Pabėgimo laikas iš pagavos centro,

$$p(\tau) = \gamma \exp(-\gamma\tau).$$

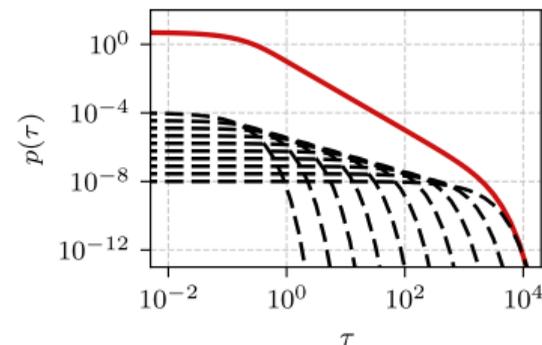
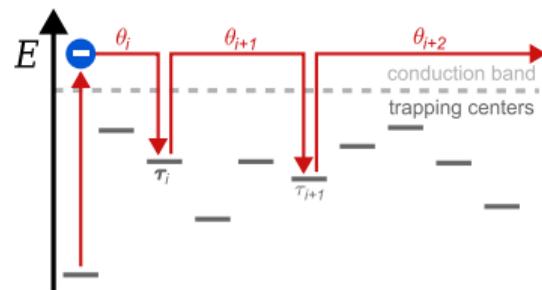
Tegu  $\gamma$  yra unikalūs, o tam tikrame intervale,

$$\gamma_{\min} < \gamma < \gamma_{\max},$$

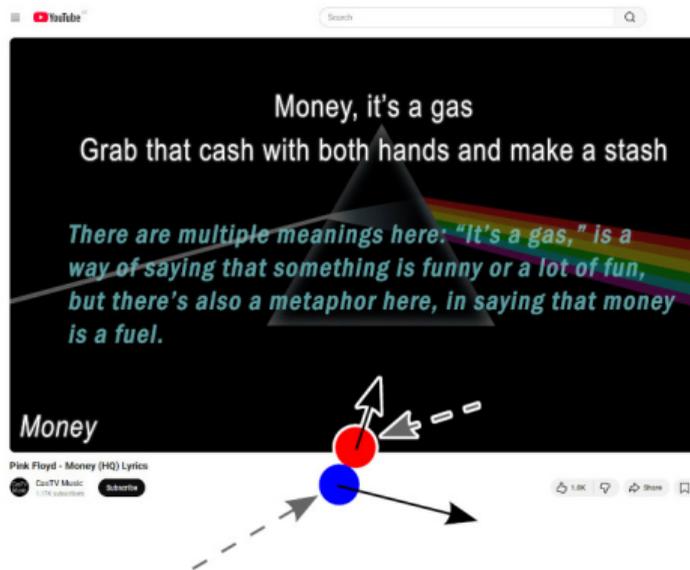
$$p(\gamma) \propto \frac{1}{\gamma^\alpha}.$$

Apjungus:

$$p(\tau) = \int_{\gamma_{\min}}^{\gamma_{\max}} p(\gamma) p(\tau|\gamma) d\gamma \propto \frac{1}{\tau^{2-\alpha}}.$$



# “Money is a gas” (Pink Floyd)



Modeliuokime turtą lyg idealias dujas:

- 1 Pasirenkame dvi daleles  $i$  ir  $j$ .
- 2 Kiek energijos bus perduota:

$$\Delta w_{ij} = (1 - \varepsilon) w_i - \varepsilon w_j,$$

čia  $\varepsilon \sim \mathcal{U}(0, 1)$ .

- 3 Atnaujiname energijas:

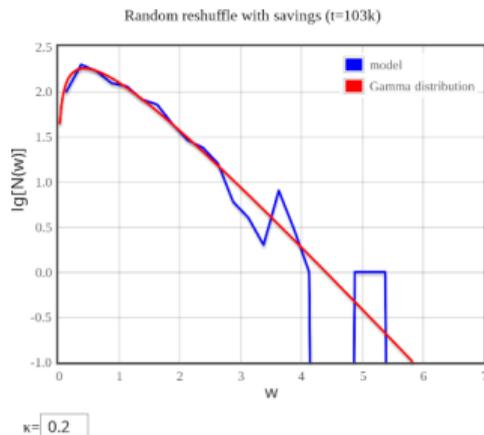
$$w_i(t+1) = w_i(t) - \Delta w_{ij},$$
$$w_j(t+1) = w_j(t) + \Delta w_{ij}.$$

# Įvairovė sukuria laipsninį skirstinį

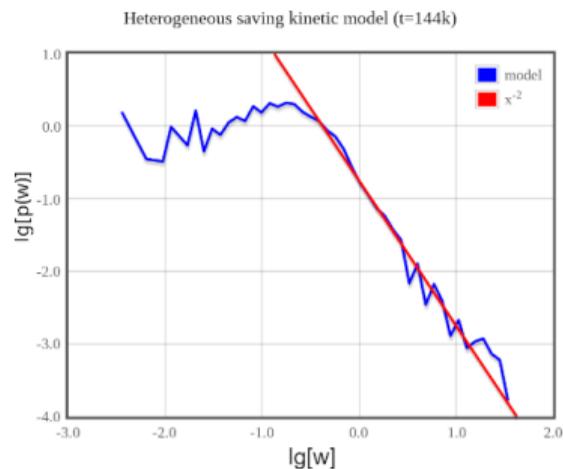
Tegu agentai pasižymi savitu taupumu  $\kappa_i$ :

$$\Delta w_{ij} = (1 - \varepsilon) (1 - \kappa_i) w_i - \varepsilon (1 - \kappa_j) w_j$$

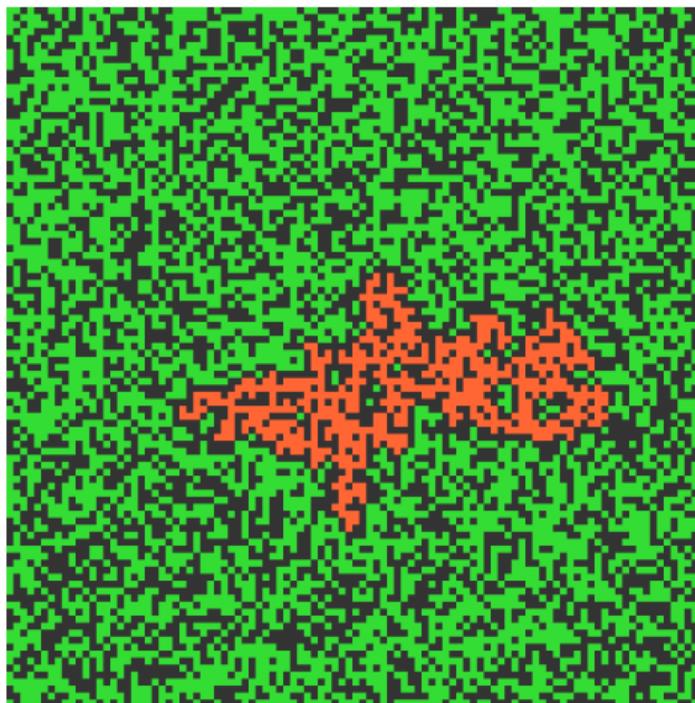
Jei  $\kappa_i = \kappa$ , tai ...



Jeigu  $\kappa_i \sim \mathcal{U}(0, 1)$ , tai ...



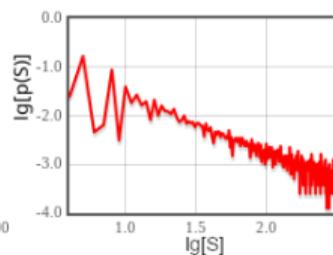
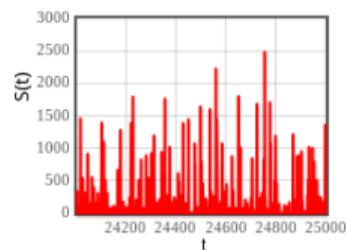
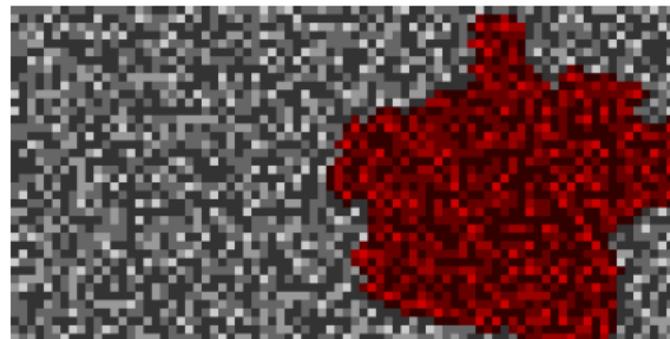
# Saviorganizuoto kritiškumo modeliai



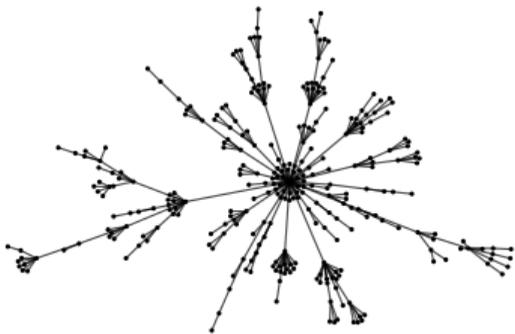
$\rho =$

Start

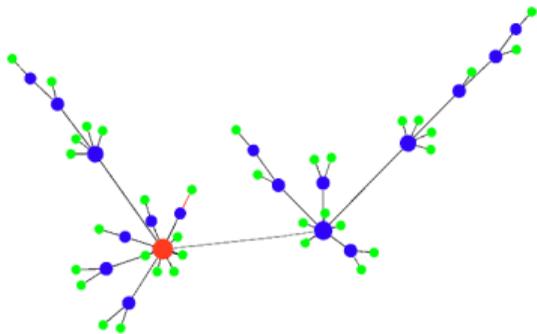
Pause



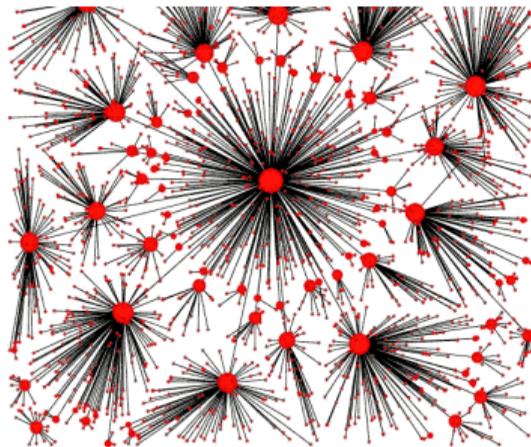
# Patys ryšiai gali būti “laipsniški”



Barabasi-Albert



Briaunos nukreipimas



Sėkmės ir priežasties

🔗 Barabasi-Albert, Edge redirection, Luck-and-reason (Physics of Risk)

# Dėkui už dėmesį!

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**Fizikos  
fakultetas**

