Gain-loss asymmetry in stock markets: Empirical facts and model based explanation

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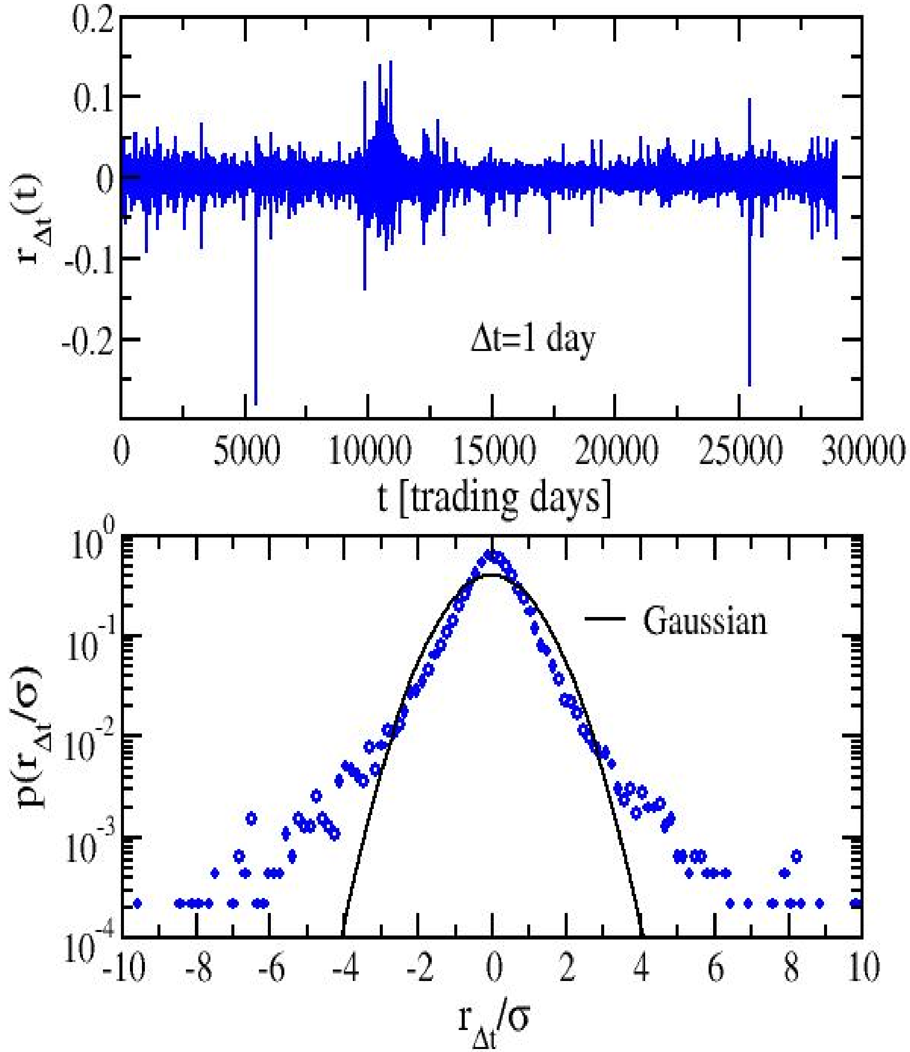
Background : The “forward” statistics

- Returns are defined as :

$$r_{\Delta t}(t) = \ln\left(\frac{S(t+\Delta t)}{S(t)}\right)$$

- Return distribution $p(r_{\Delta t})$
  - *Fixed* time interval Δt used
- Stylized facts:
  - Close to symmetric distribution
  - Slow convergence towards a Gaussian distribution
  - Fat tails

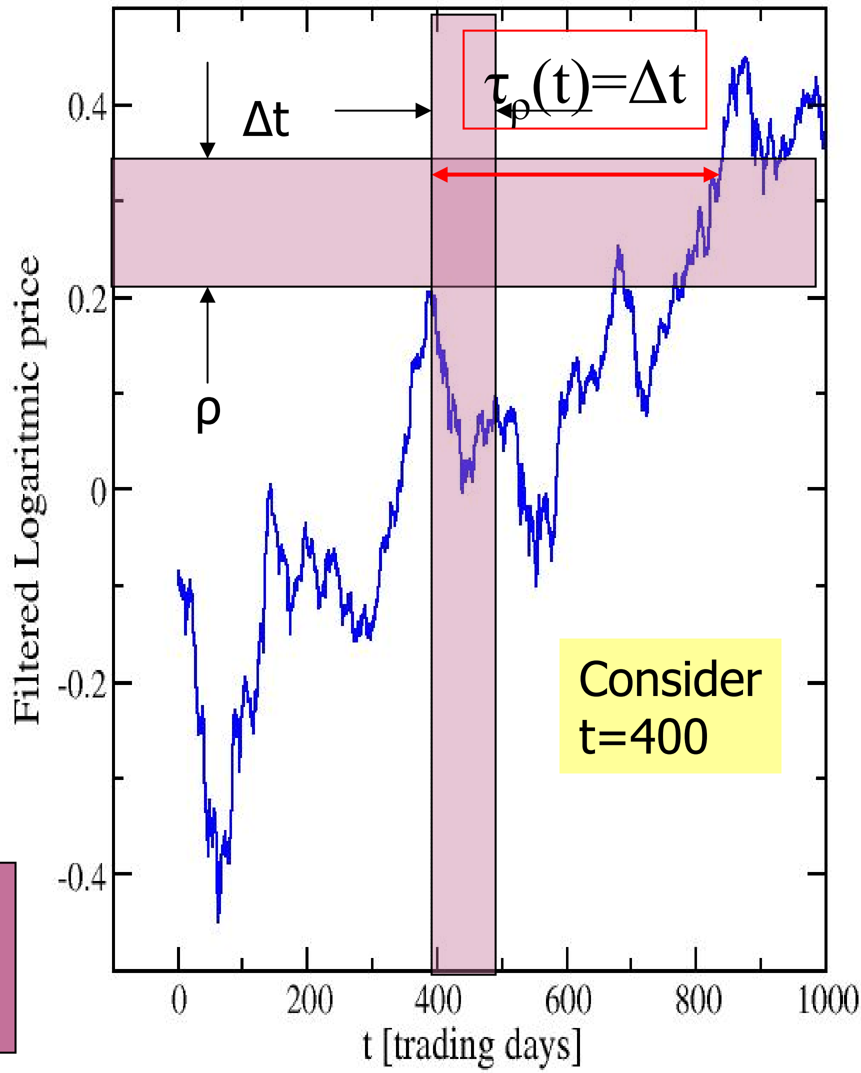
DJIA (1896-2001)



$p(r_{\Delta t})$ is classically used to gauge asset performance

The Concept of Inverse Statistics

- PDF of returns
  - Fixed time window Δt
- Inverse statistics
  - Fix the return level $r_{\Delta t}(t)=\rho$!
  - Waiting time distribution (WTD)
  - For time t, monitor the level of return for increasing Δt until $r_{\Delta t}(t) \geq \rho$ for the first time.
  - Eur. J. Phys. B 27, 583 (2002)



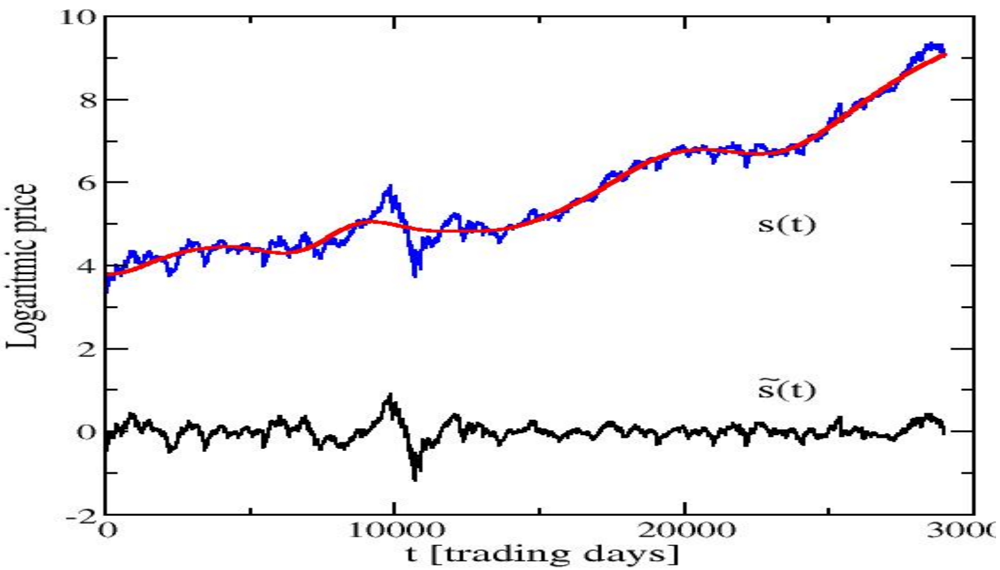
Investment Horizon : $\tau_\rho(t)$

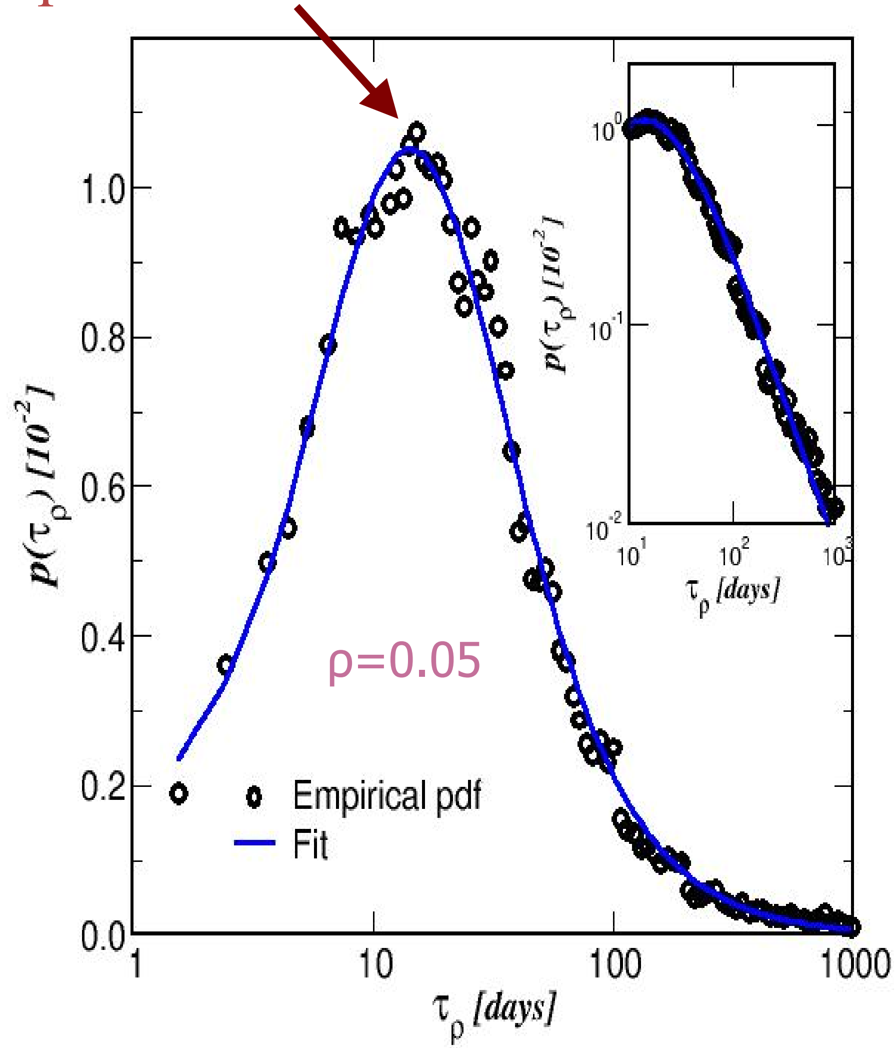
Waiting time distribution : $p(\tau_\rho)$

DJIA (1896-2001)

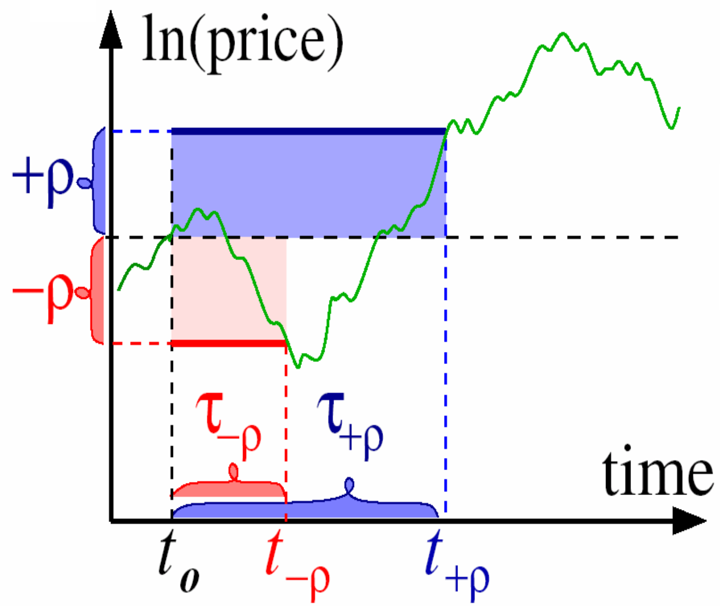
- Remove trends
- Data well fitted by a generalized inverse Gamma-distribution

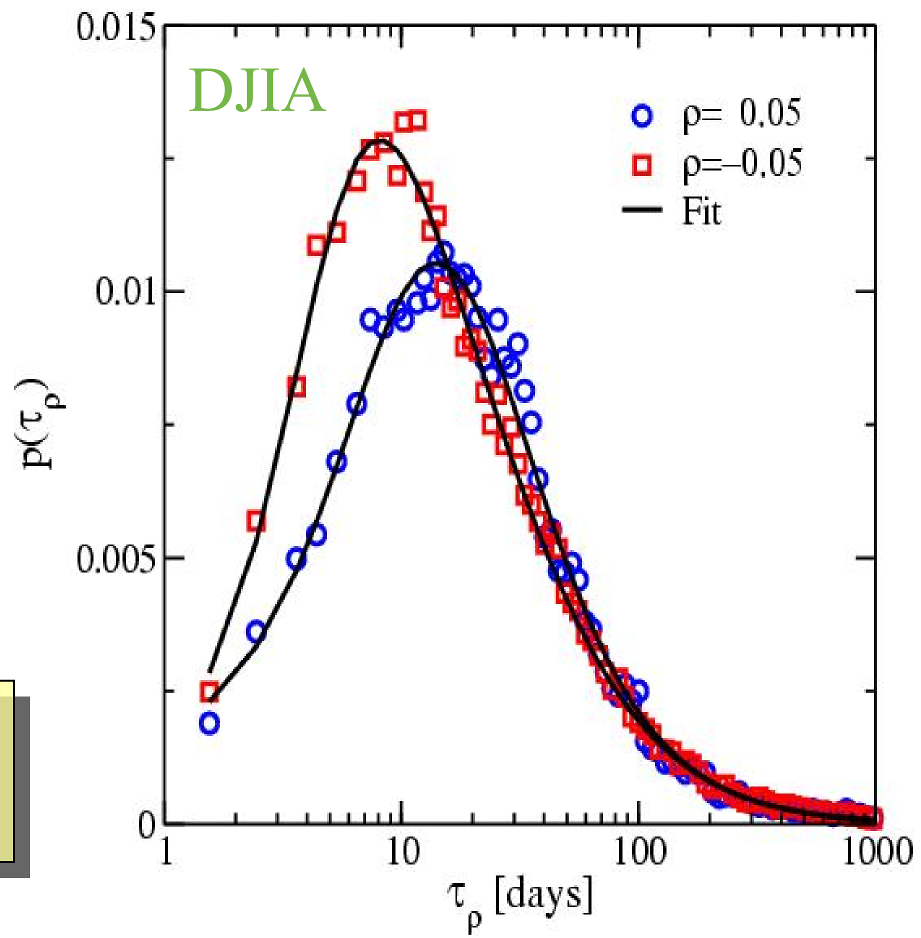
$$p(t) = \frac{\nu}{\Gamma\left(\frac{\alpha}{\nu}\right)} \frac{\beta^{2\alpha}}{(t+t_0)^{\alpha+1}} \exp\left[-\left(\frac{\beta^2}{t+t_0}\right)^{\nu}\right]$$





Empirical findings: Indices



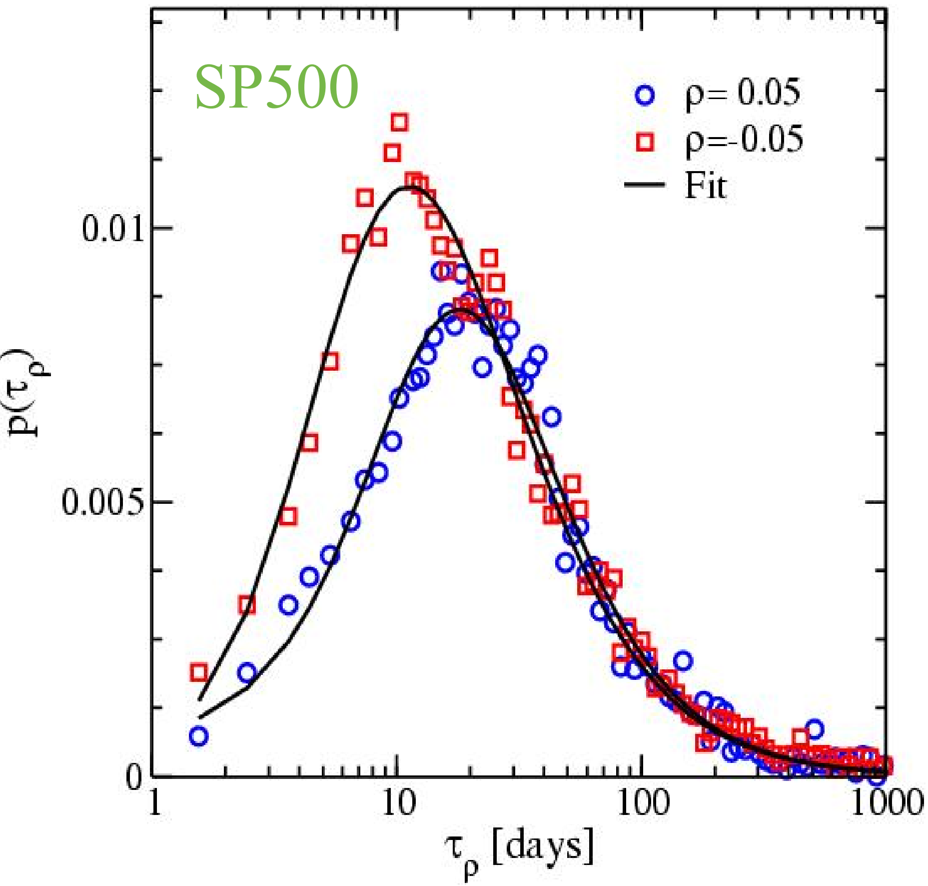


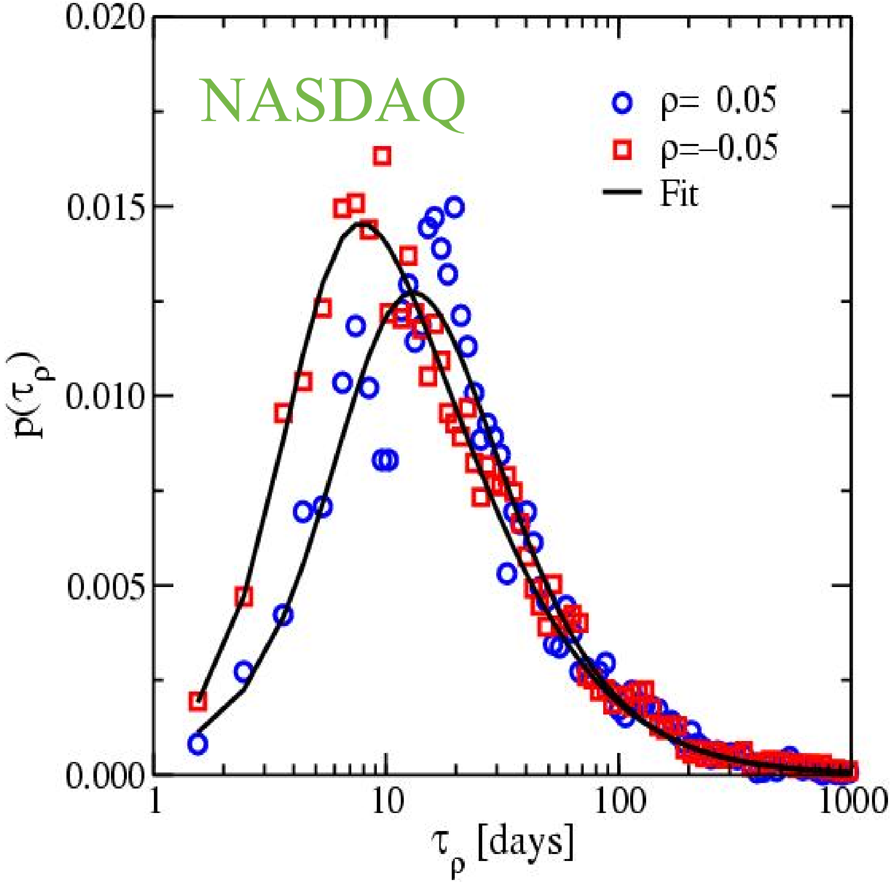
NOTE the asymmetry!

Losses are faster than *gains.*

Empirical findings: Indices

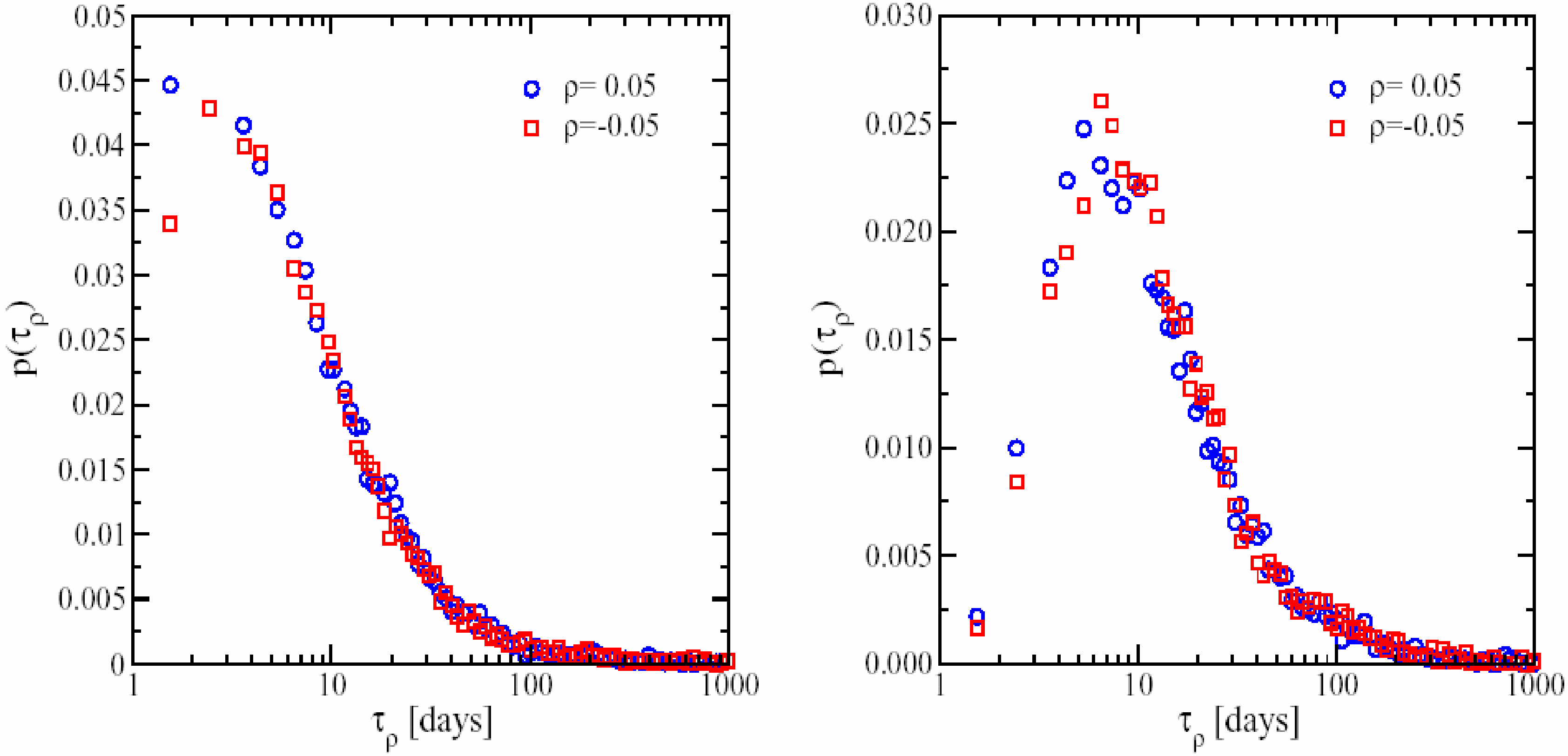
Not only the DJIA is asymmetric!





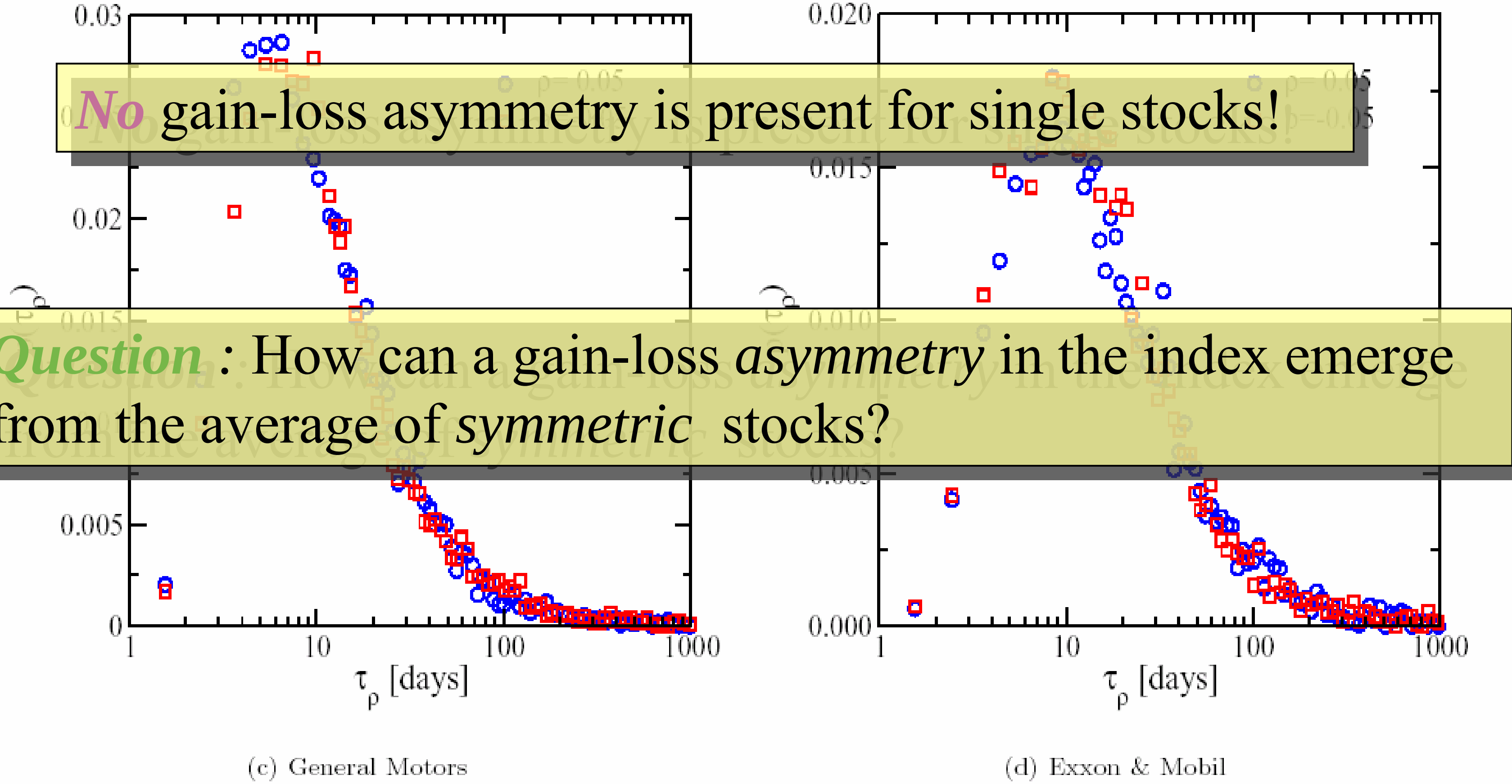
Empirical findings: …*but*, single stocks..

Some of the stocks constituting the DJIA!



(a) Boeing Airways (b) General Electric

Empirical findings: …*but*, single stocks..

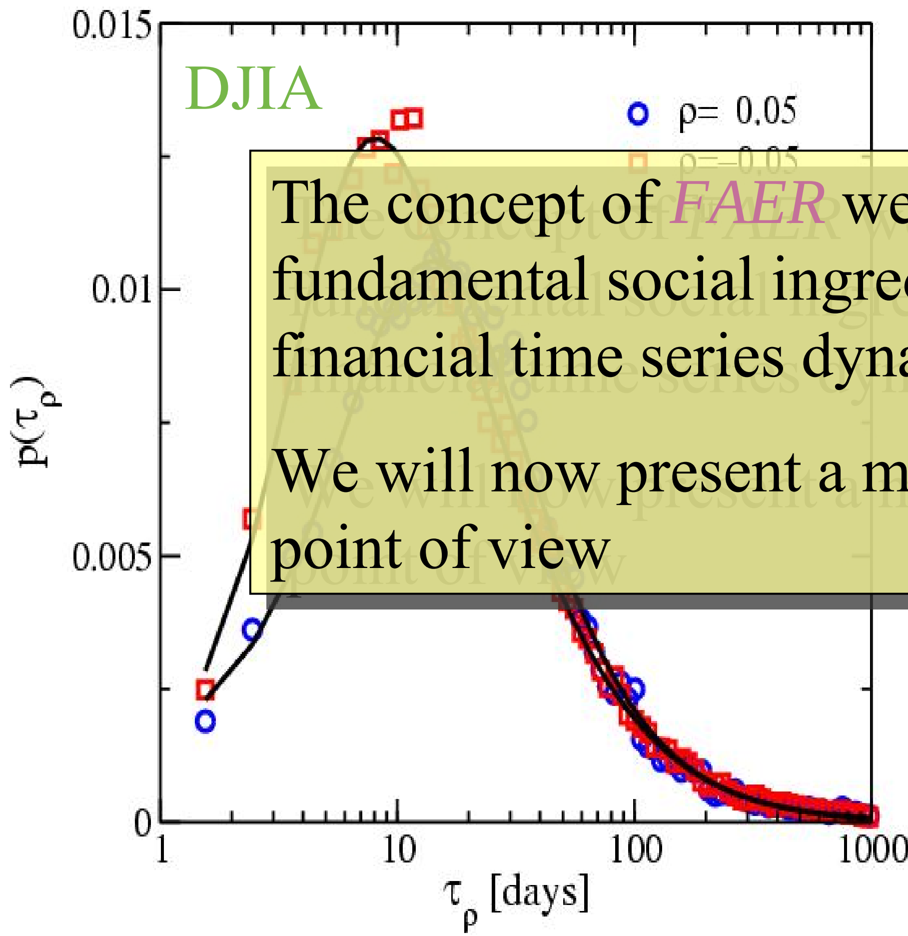


No gain-loss asymmetry is present for single stocks!

Question : How can a gain-loss *asymmetry* in the index emerge from the average of *symmetric* stocks?

FIG. 7: Same as Fig. 3(a), but for some of the individual companies of the DJIA: (a) Boeing Airways (1962.1–1999.8); (b) General Electric (1970.0–1999.8); (c) General Motors (1970.0–1999.8); (d) Exxon & Mobil, former Standard Oil (1970.0–1999.8).

How to rationalize these findings?



- Under up-trends stocks move more or less randomly
- Psychology of society/market: When stocks begin to fall they do it synchronously
- Traders are more sensitive to negative than to positive information
- External events (wars, terror, earthquakes, hurricanes) introduces ***fear*** into the market place

The concept of *FAER* we think is an important fundamental social ingredient for understanding financial time series dynamics.

We will now present a model illustrating our point of view

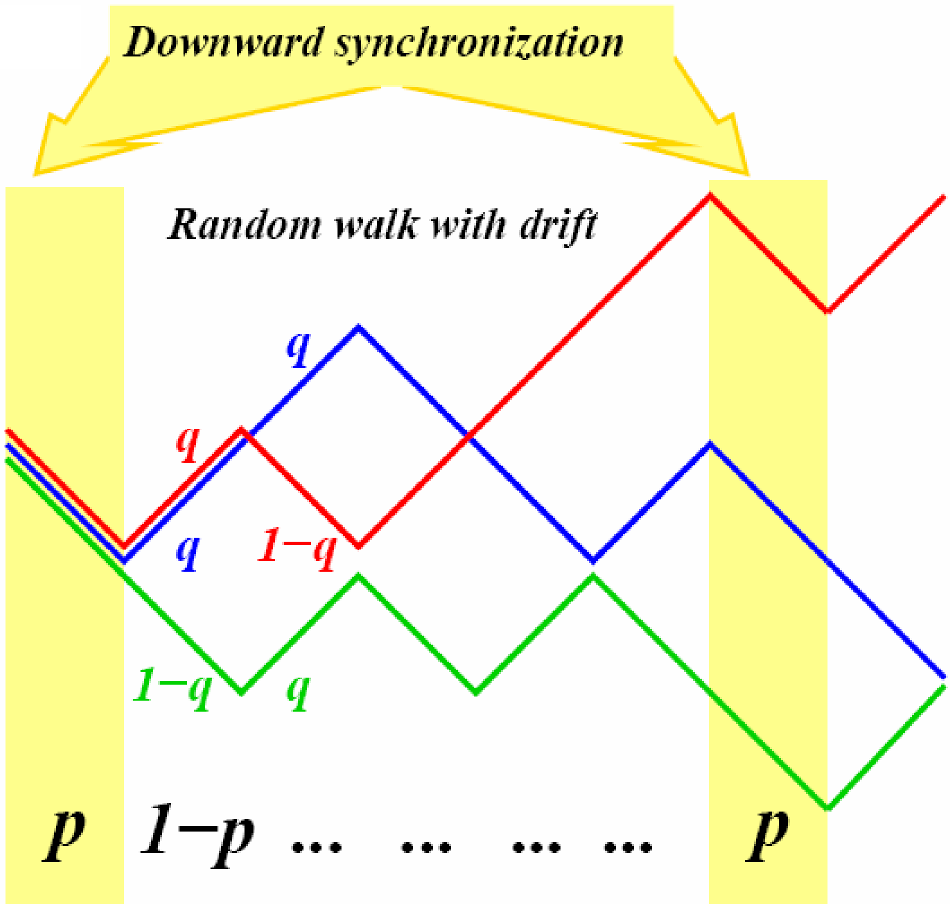
The Fear Factor Model: The assumptions

- Single Stocks
  - The price process of a single stock, S(t), makes a *Geometrical Brownian Motion:*
    - i.e. $s(t)=\ln[S(t)]$ is Brownian random process
  - s(t) is un-biased (no drift)

- The Stock Index
  - The stock index consists of N stocks
  - The value of the stock index, I(t), is calculated as:

$$I(t)=\sum_{i=1}^{N} S_i(t)=\sum_{i=1}^{N}\exp\left(s_i(t)\right), \qquad S_i(t)=\ln s_i(t)$$

The Fear Factor Model (FFM)



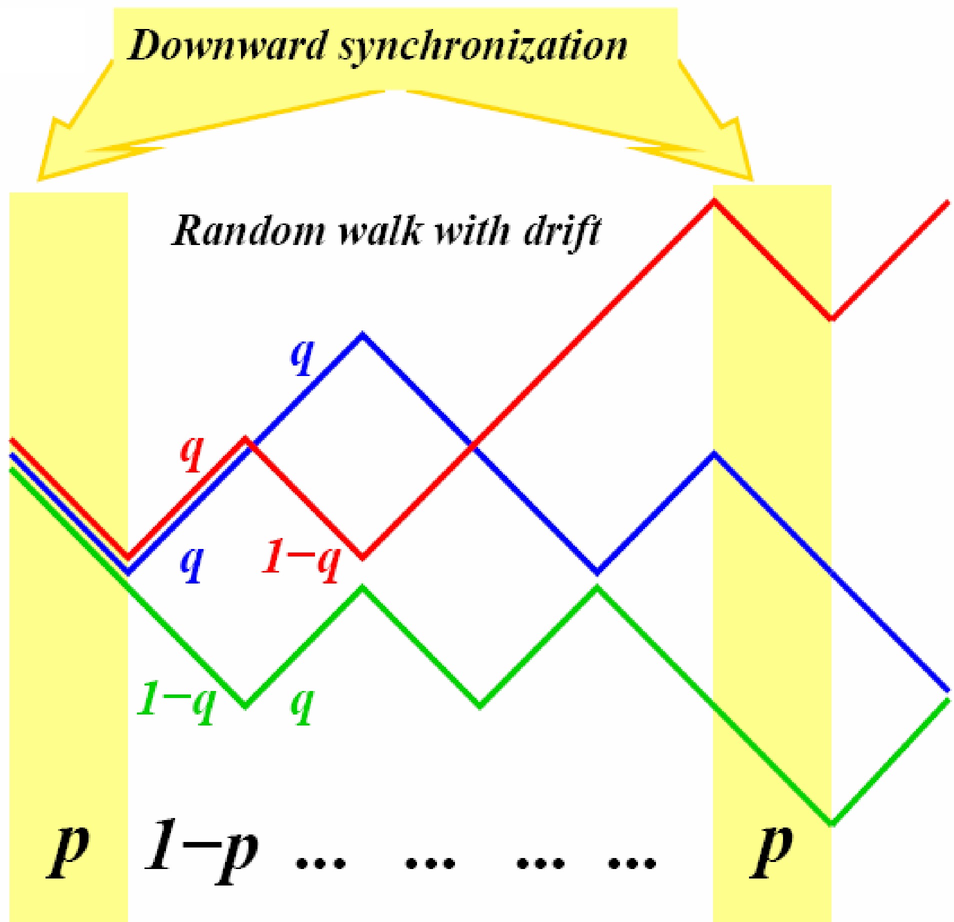
For the log-price of a stock:

- With prob. p : *all* stocks move *downwards synchronously*
- With prob. 1-p : they do *independent biased* random walks
  - With prob. q : move upward
  - With prob. 1-q : move downward
- q determined from:
  - Requirement : $s_i(t)$ is drift-less

p : fear factor

N : # of stocks in the index

The Fear Factor Model (FFM)



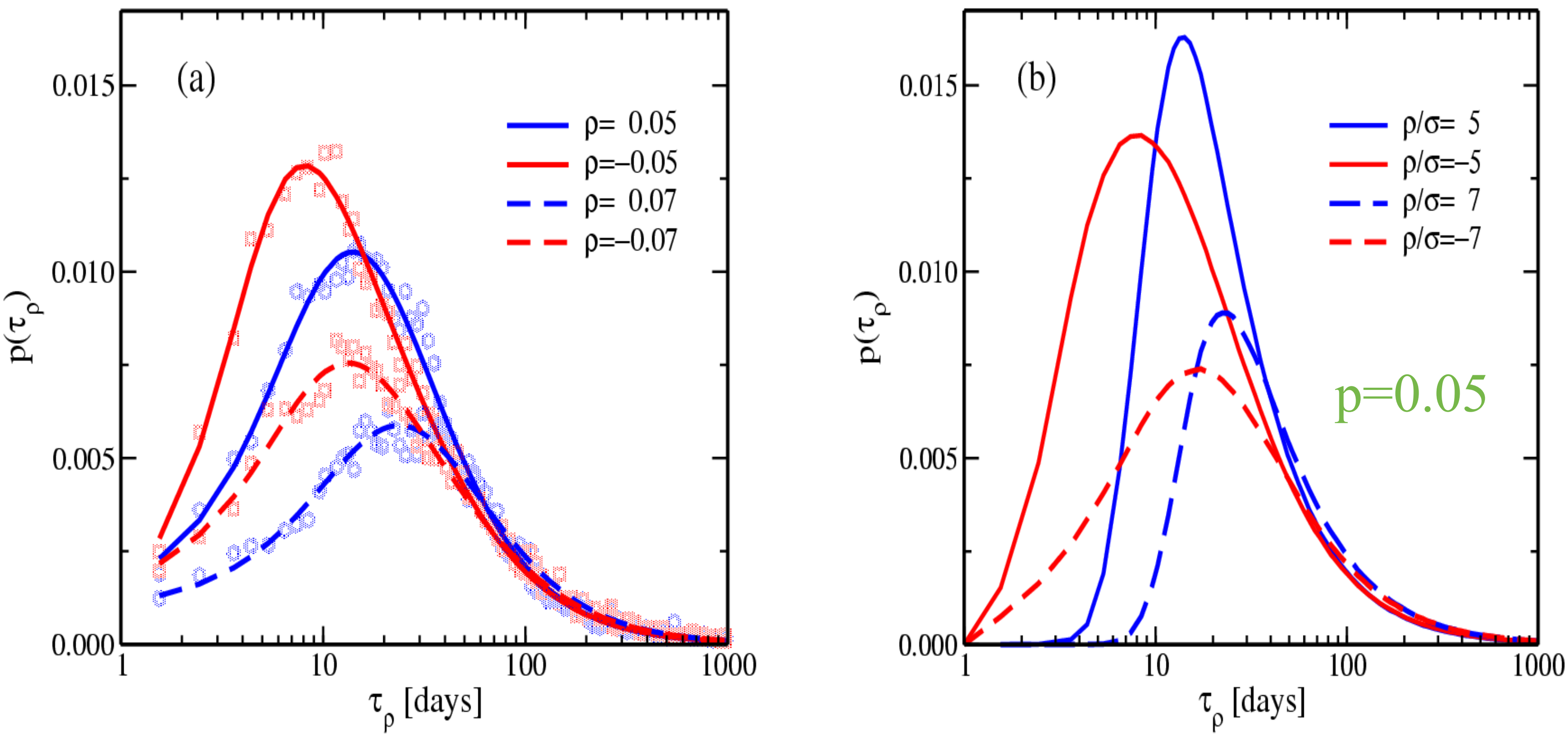
The q-parameter is determined from the assumption: *indiv. stock prices are drift-less*

- $\underbrace{p + (1-p)(1-q)}_{\text{Price-drop}} = \underbrace{(1-p)q}_{\text{Price-rise}}$
- p and q are coupled
- $$q = \frac{1}{2(1-p)}$$

Model results

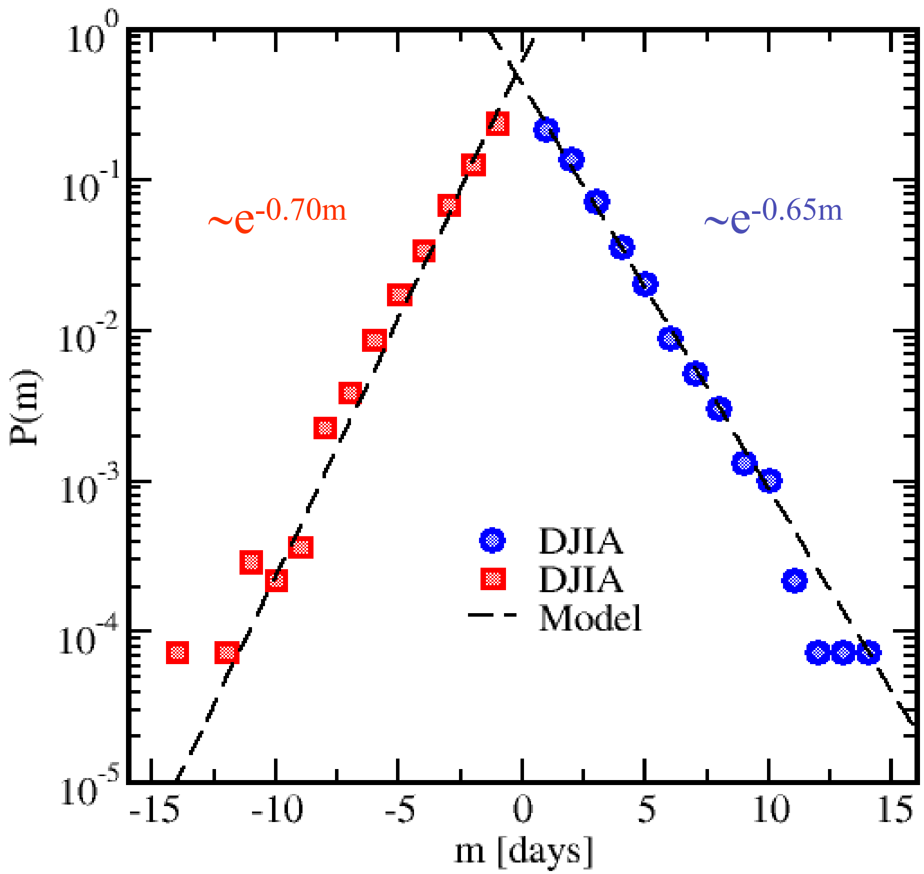
Empirical (DJIA)

Model (σ :volatility)



Model results

NOTE the slight asymmetry



Let us consider the probability that the DJIA index drops (m<0) or rises (m>0) several days (*m*) in a row ("mini crashes/rallies")

m=1 : 10% more likely to have a price drop than a price rise

The model catch *also* this feature of the real market excellently!

Conclusions

- *Fear* might be a fundamental social ingredient for financial time series dynamics
- Stock index data typically show a gain-loss asymmetry, while individual stocks do not (*new stylized fact*)
- *Synchronous draw-downs* seems to play a role for stock indices
- The *Fear Factor Model* nicely reproduces empirical results

References

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Thank you for your attention!

No Commutation (Avr./Inv.Stat.)

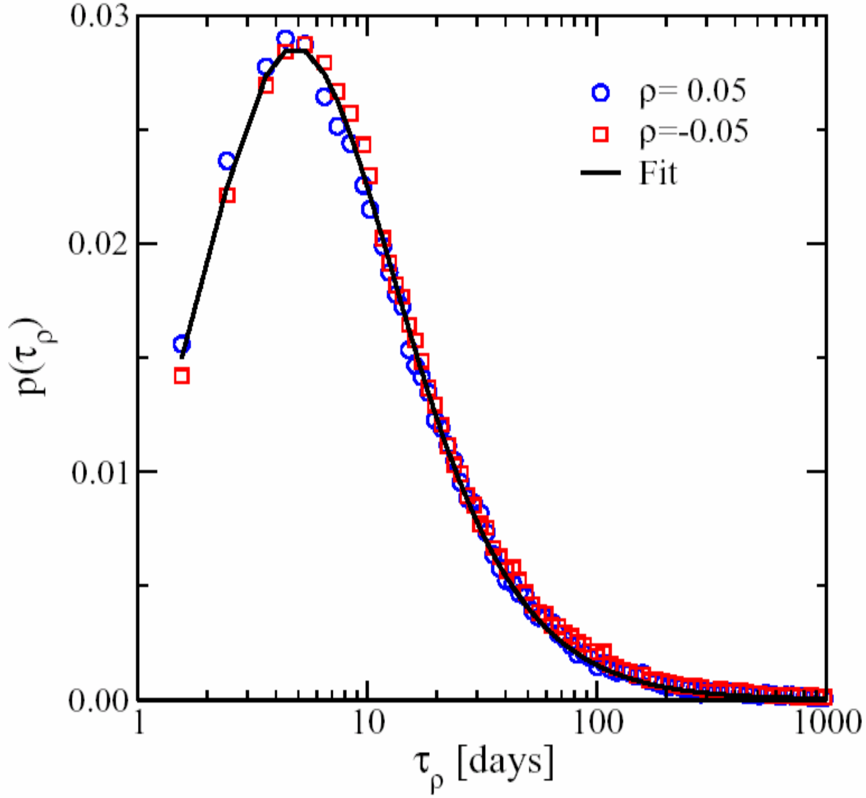

FIG. 8: Averaged gain and loss distribution for the companies listed in table I. The fit is Eq. (2) with values $\alpha \approx 0.60$, $\beta \approx 3.24$, $\nu \approx 0.94$ and $t_0 \approx 1.09$. Note that the tail exponent $\alpha + 1$ is 0.1 above the "random walk value" of 3/2.