Critical Exponents of 3D Ising Model from Theory and Monte Carlo Simulations of Very Large Lattices

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Reorganized perturbation theory

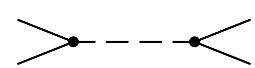
(Ann. Phys. (Leipzig) 10 (2001) 299)

Consider φ^4 model with the Hamiltonian

$$H/T = \int \big(r_0 \varphi^2(\mathbf{x}) + c(\nabla \varphi(\mathbf{x}))^2 + u\varphi^4(\mathbf{x}) \big)\, d\mathbf{x} \ ,$$

Grouping of Feynman diagrams $\Rightarrow$ the Dyson equation

$$\frac{1}{2G_i(\mathbf{k})} = r_0 + ck^2 - \frac{\partial D(G)}{\partial G_i(\mathbf{k})} + \vartheta_i(\mathbf{k})$$

for the correlation function $\langle \varphi_i(\mathbf{k})\, \varphi_j(-\mathbf{k}) \rangle = \delta_{ij}\, G_i(\mathbf{k})$. Here $D(G)$ is the (resummed) sum of grouped *skeleton diagrams* constructed of the fourth order vertices  , including *all* original diagrams of φ^4 perturbations.

Advantages

- The method allows to make certain analysis without cutting the series $\Longrightarrow$ exact critical exponents.
- The asymptotics of $G(\mathbf{k})$ is found *directly as an expansion in powers of* k avoiding doubtful intermediate expansions in divergent parameters like $\ln k$.

The latter problem is not satisfactory solved in the perturbative RG approach. $\ln k$ diverges at $k \to 0$ and the RG method is not correct, since a contradiction can be derived! – Sec. 2 in Ann. Phys. (Leipzig) 10 (2001) 299. 1) correction-to-scaling for $1/[k^2 G(\mathbf{k})]$ is $\delta X(\mathbf{k}, \mu) = \mathcal{O}(\epsilon^2)$, as obtained from a first-principles equation assuming the Wilson–Fisher fixed point; 2) we get $\delta X(\mathbf{k}, \mu) = \mathcal{O}(\epsilon)$ by matching coefficients at $\ln k$, since $\omega = \epsilon + \mathcal{O}(\epsilon^2)$.

Critical exponents

Critical exponents predicted by grouping of Feynman diagrams: for $n = 1, 2, \ldots$, $d < 4$ (only $n = 1$ for $d = 2$)

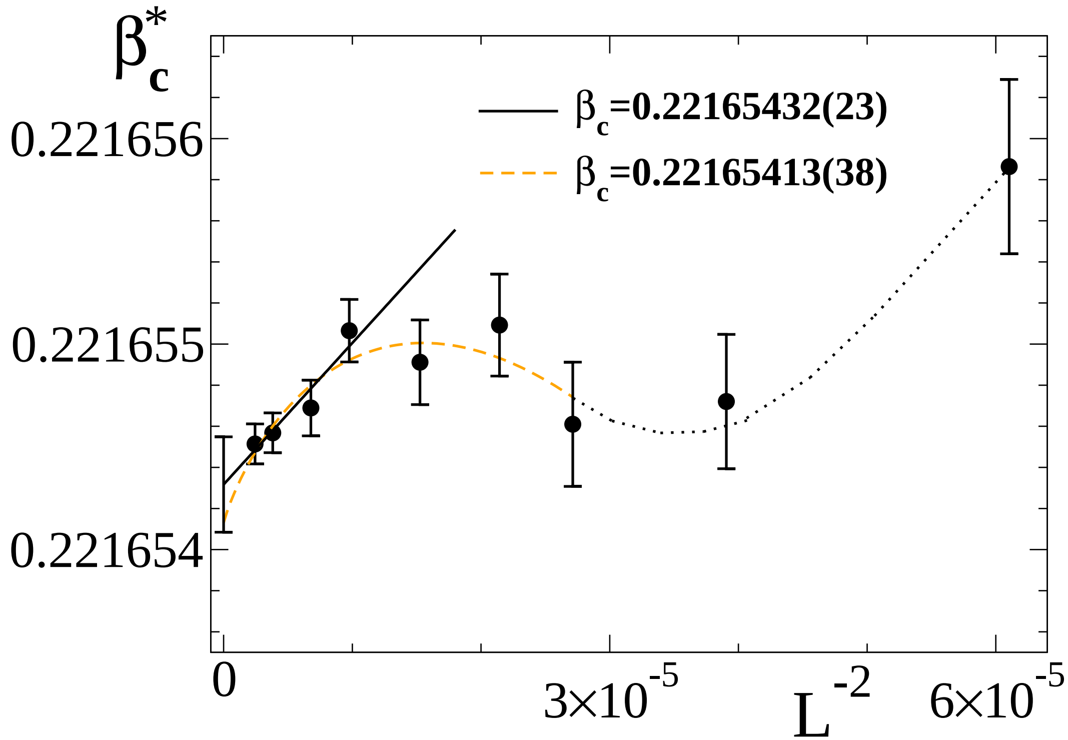
$$\gamma = \frac{d + 2j + 4m}{d(1 + m + j) - 2j} \qquad \nu = \frac{2(1 + m) + j}{d(1 + m + j) - 2j},$$

where $m \geq 1$ and $j \geq -m$ are integers. It reproduces the know exact critical exponents of 2D Ising model ($m = 3$, $j = 0$), the mean–field exponents at $d \to 4$ (any j and m), as well as those of the spherical model ($j/m \to \infty$).

The Ising case: $m = 3$, $j = 0 \Rightarrow \gamma = 7/4$, $\nu = 1$ at $d = 2$ and $\gamma = 5/4$, $\nu = 2/3$ ($\alpha = 0$, $\eta = 1/8$, $\beta = 3/8$) at $d = 3$.

The conventional (RG) values: $\gamma \simeq 1.24$, $\nu \simeq 0.63$, $\alpha \simeq 0.11$ $\eta \simeq 0.0335$, $\beta \simeq 0.326$.

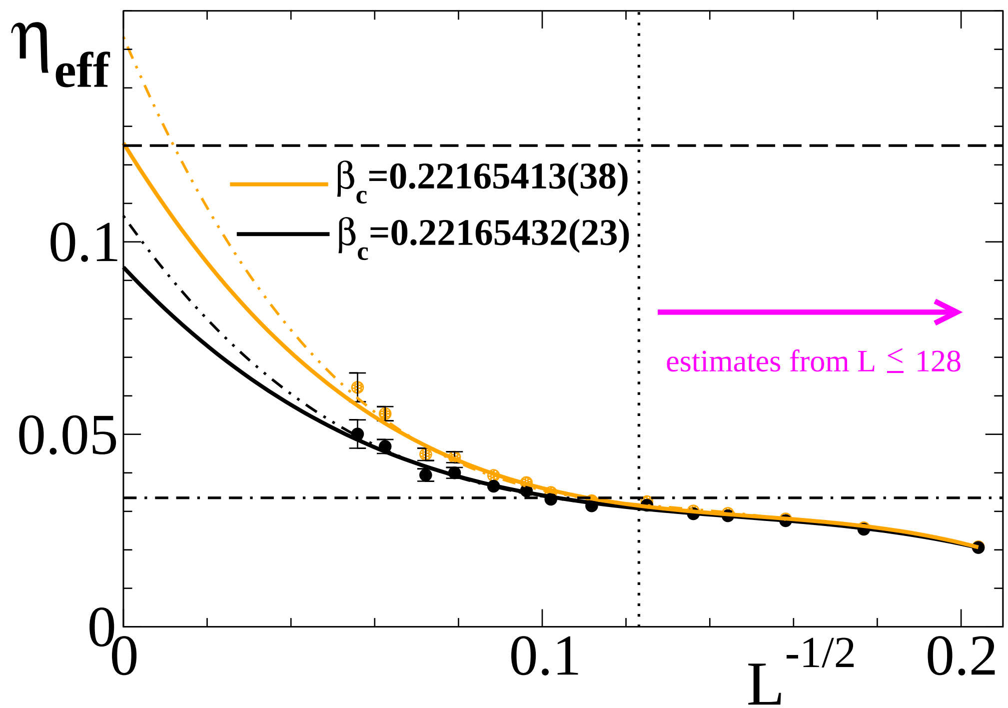
Estimation of the critical coupling



Couplings $\beta_c^*(L)$ ($\approx$ Binder cumulant crossing points) obtained from $\tilde{\beta}_c(L_1)$ data at $L_1 = L, L/2$ with $128 \leq L \leq 640$, where $\tilde{\beta}_c(L)$ correspond to $U = \langle m^4 \rangle / \langle m^2 \rangle^2 = 1.6$. The $L^{-(1/\nu)-\omega}$ convergence to β_c is expected. Estimation with $\nu = 2/3$ and $\omega = 1/2$ (shown) yields $\beta_c = 0.22165432(23)$ (linear fit) and $\beta_c = 0.22165413(38)$ (nonlinear fit).

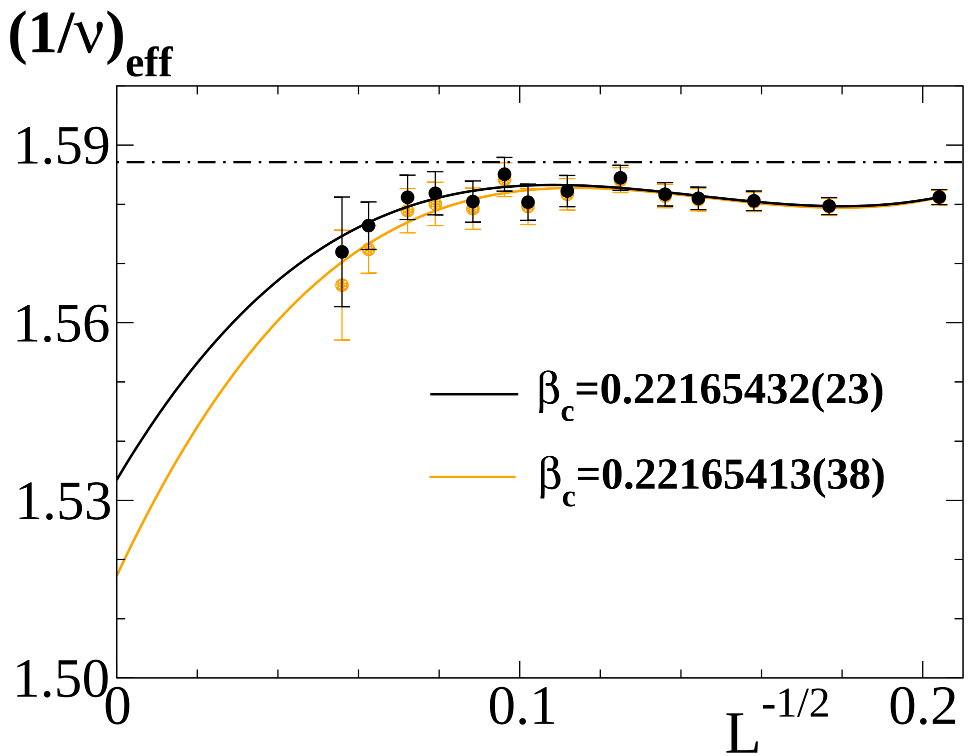
$\nu = 0.63$ and $\omega = 0.8$ gives $\beta_c = 0.22165438(18)$ and $\beta_c = 0.22165429(25)$, respectively.

Estimation of η in 3D Ising model



The effective critical exponent η_{eff} vs $L^{-1/2}$ estimated from susceptibility data at $\beta = \beta_c$ for each pair of sizes $(2L; L/2)$ according to $\chi \sim L^{2-\eta}$. The simulated range of sizes $[12; 640]$. Our theoretical value $\eta = 1/8$ (dashed line), the RG value $\eta \simeq 0.0335$ (dot-dashed line).

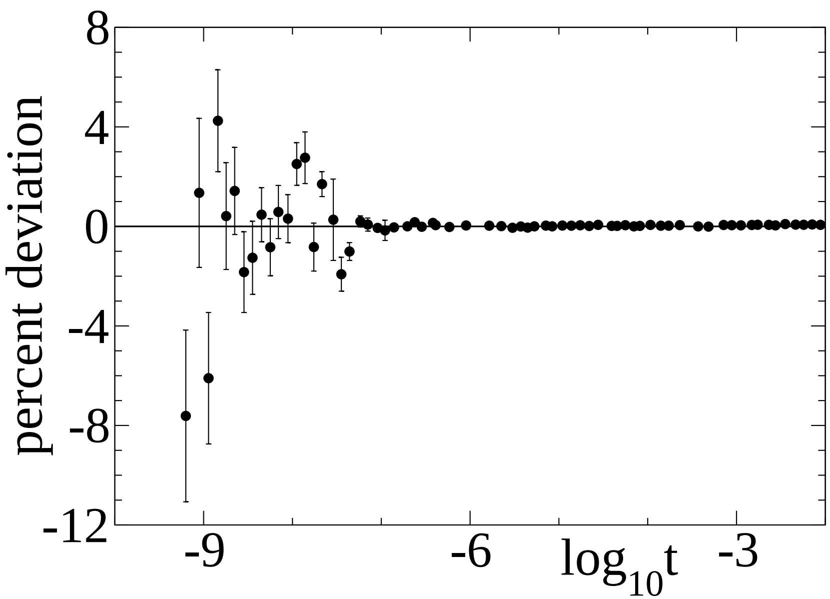
Estimation of ν in 3D Ising model

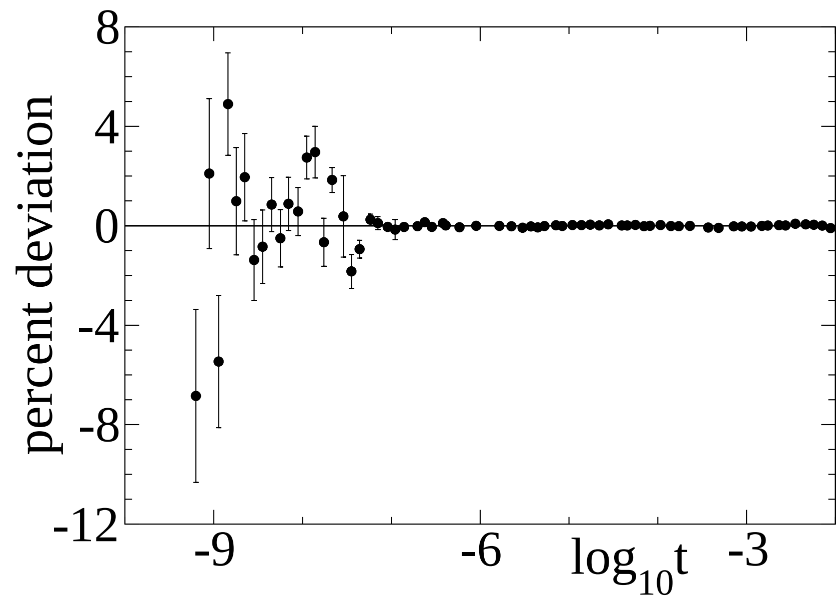


The effective critical exponent $(1/\nu)_{\text{eff}}$ vs $L^{-1/2}$ estimated from the derivative of $\langle m^2\rangle^2/\langle m^4\rangle$ at $\beta = \beta_c$ (with $\sim L^{1/\nu}$ scaling) for each pair of sizes $(2L; L/2)$. The simulated range $[12; 640]$. Our theoretical value $1/\nu = 1.5$, the RG value $1/\nu \simeq 1.587$ (dot-dashed line).

Comparison to C_p data in liquid helium

(following experiments by J. A. Lipa, et. al, see Phys. Rev. B **68**, 174518 (2003))



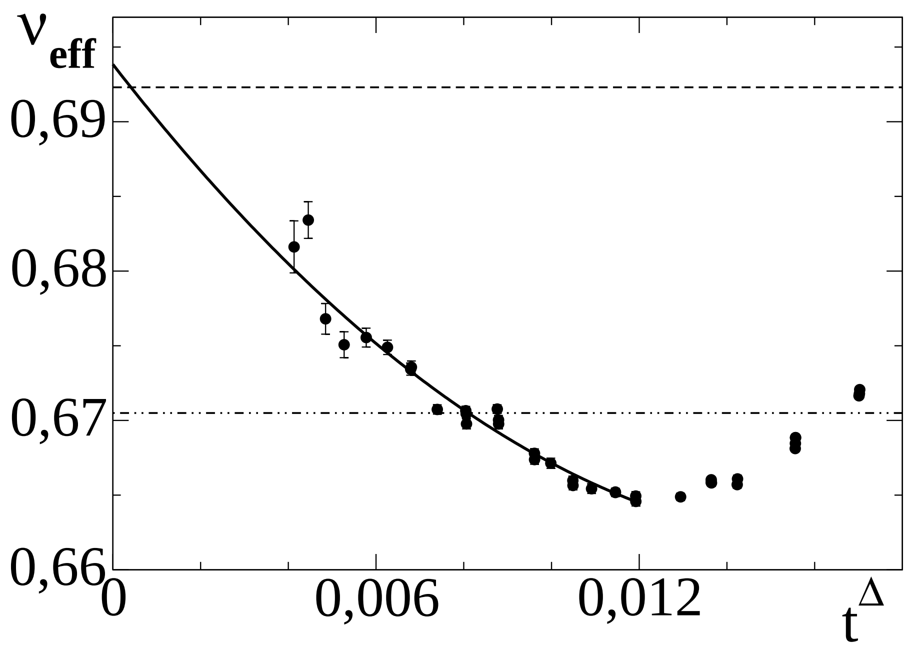


Eur. Phys. J. B **45**, 459 (2005): percent deviation from

$$C_p = \frac{A}{\alpha} t^{-\alpha} \left(1 + at^{\Delta} + bt^{2\Delta}\right) + B,\ \ \alpha = -0.0127, \Delta = 0.529$$ **(left)**

$$C_p = t^{-\alpha}(C + A \ln t) \left(1 + at^{\Delta}\right) + B,\ \ \alpha = -1/13, \Delta = 5/13$$ **(right)**

ν from superfluid fraction in liquid He



Eur. Phys. J. B **45**, 459 (2005): The effective exponent ν_{eff} depending on t^{Δ} (with $\Delta = 5/13$) evaluated from local slopes of $\ln \rho$ vs. $\ln t$ plot. The lower line – (RG) value 0.6705, the upper line – our theoretical value $\nu = 9/13$.

Data taken from L. S. Goldner, N. Mulders, G. Ahlers, J. Low Temperature Phys. **93**, 131 (1993).