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Individual decisions under risk: an investigation of observational economics

Overview

- Introduction to the topic of decision making under risk

- An experiment of observational economics

Utility function

The concept of utility function dates back to Daniel Bernoulli†

†D. Bernoulli, Exposition of a new theory on the measurement of risk, Papers of the Imperial Academy of Sciences in Petersburg, vol 5, 1738 Translated from Latin into English by Dr. Louise Sommer, Econometrica 22, 23-36 (1954).

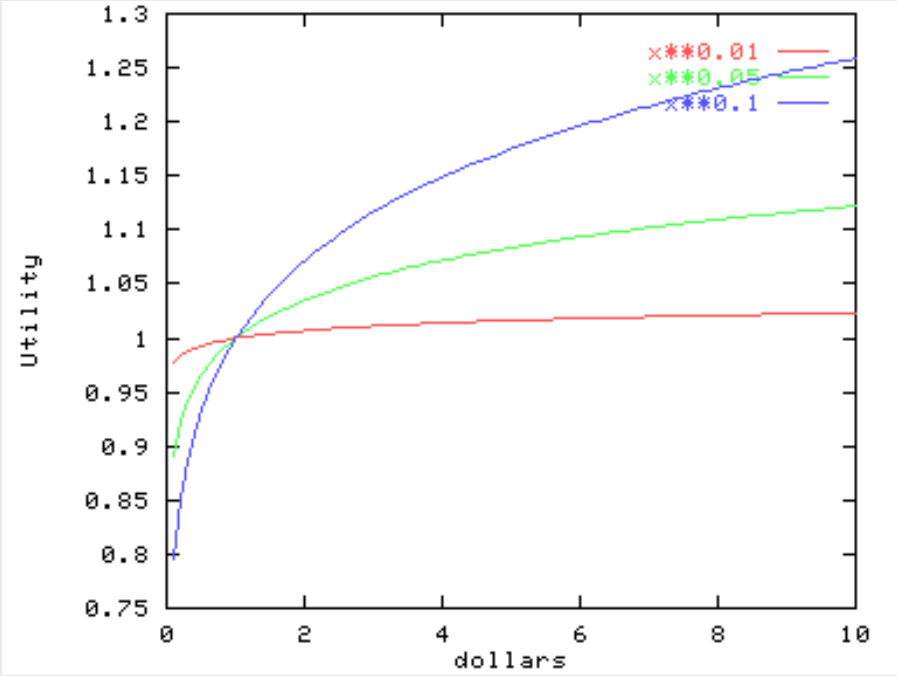
Utility function

Utility function comes back into Economics in 1947 thanks to J. von Neumann and O. Morgenstern

"Any rational decision-maker's behavior should be describable by a utility function, which gives a quantitative characterization of his preferences for outcomes or prizes, and a subjective probability distribution, which characterizes his beliefs about all relevant unknown factors"

A key property of utility function

A decision maker is globally risk averse if and only if his von Neumann-Morgerstern utility function of wealth is strictly concave at the relevant wealth levels

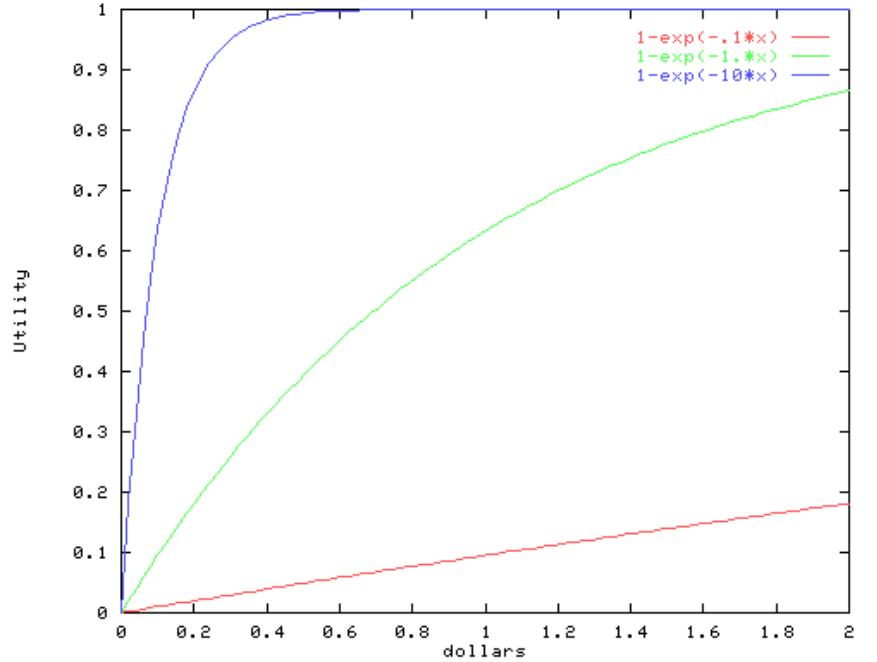


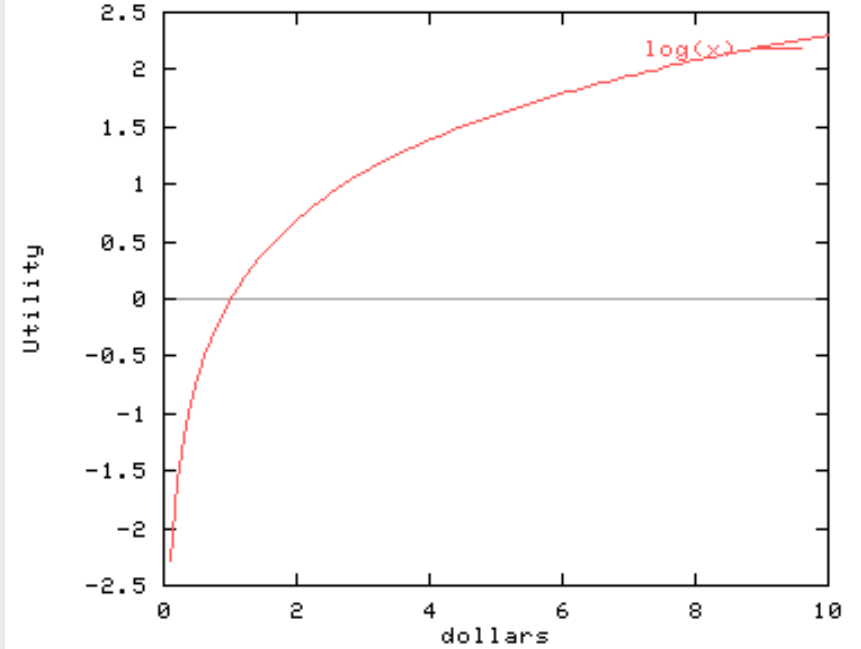
Historical remarks

Already in Bernoulli original article there is a key consideration about risk aversion.

"Somehow a very poor fellow obtains a lottery ticket that will yield with equal probability either nothing or twenty thousands ducats. Will this man evaluate his chances of winning at ten thousands ducats? Would he not be ill-advised to sell this lottery ticket for nine ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousands ducats ..... to do this the determination of the value of an item must not be based on its price, but rather on the utility it yields."

Some used utility functions

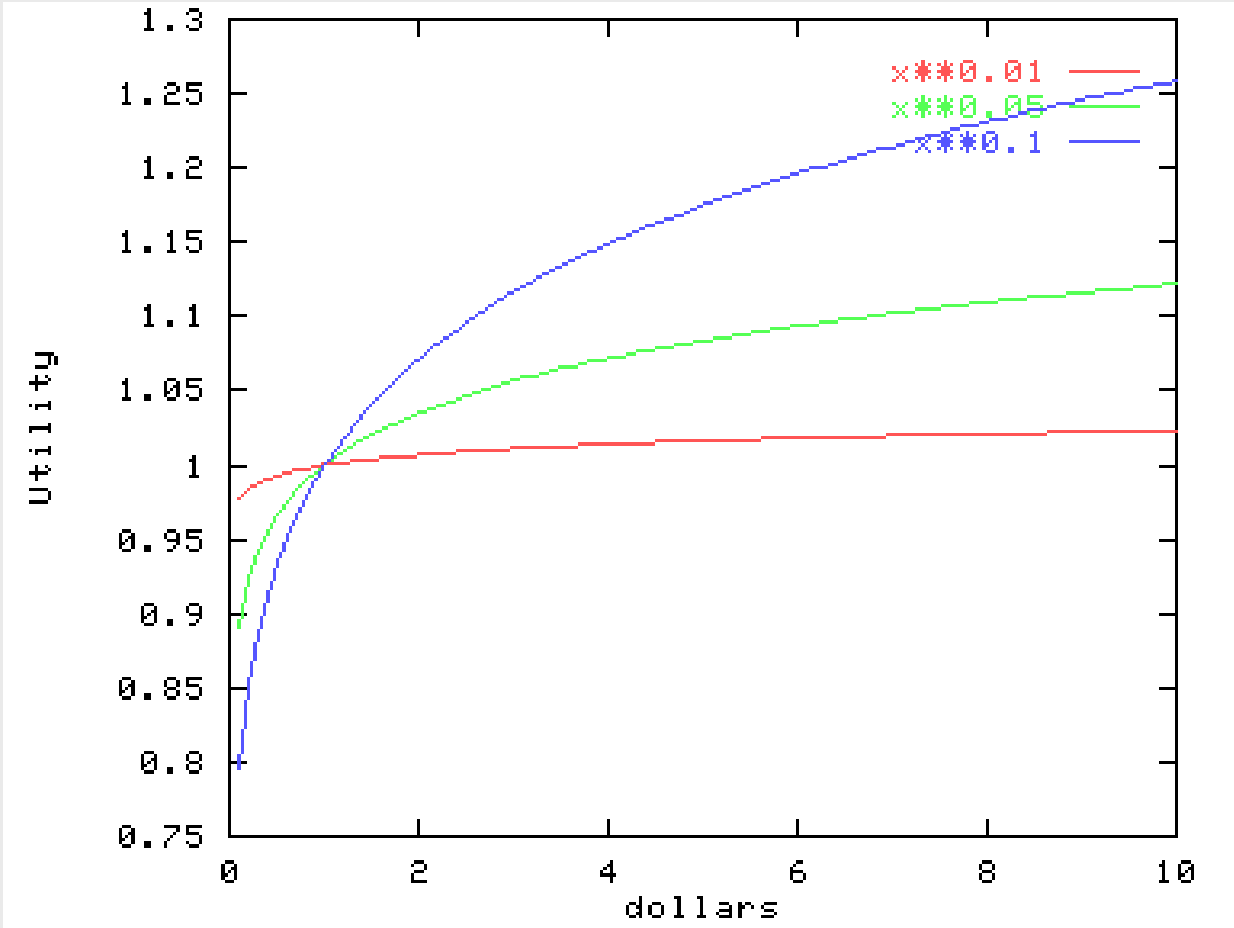




$$U(x) = 1 - \exp(-kx)$$

$$U(x) = \log(x)$$

A widely used utility function



$$U(x) = cx^{\gamma}$$

Eliciting utility functions

There is a huge economic literature on the elicitation of utility function in economics.

Empirical studies have been performed in a variety of fields ranging from lottery gambles to medical problems

In most cases empirical studies are devised by proposing a potential situation to a decision maker (typically a student) and asking her/him about the decision she/he would take. For such collaboration, the decision maker is usually taking a certain amount of money

Field investigations

Sometime, experiments are performed in the field, usually in a rather poor environment and are conducted with real money. the amount of money might be significant for the participants. Examples are investigations performed in rural India and Bazil.

A prospect

In typical investigations a decision maker faces something like the following situation:

Choose between:

A: 4,000 with probability 0.8

or

B: 3,000 with probability 1.0

Individual decisions under risk

In an investigation†

Choose between:

A: 4,000
with probability 0.8

or

B: 3,000
with probability 1.0

N=95

[20%]

[80%]

Note that:

$$E\{\text{Win}\} = 3{,}200$$

$$E\{\text{Win}\} = 3{,}000$$

†Kahneman and Tversky, 1979

This result shows risk aversion.

Violation of expected utility theory

Things are indeed more complicated that just that, and in fact Kahneman and Tversky have shown that expected utility theory may be violated

Kahneman and Tversky experiment

The two prospects:

A: 4,000 with prob. 0.80 [20%]

B: 3,000 with prob. 1 [80%]

C: 4,000 with prob. 0.20 [65%]

D: 3,000 with prob. 0.25 [35%]

are not perceived as equivalent by the decision makers in spite of the fact that

C(4000,0.20)=(A,0.25)

D(3000,0.25)=(B,0.25)

Individual decisions under risk

In summary, beautiful results about expected utility theory and prospect theory are present in the economic literature. Most of the results are obtained in an (economic) laboratory.

The aim of the present work is to obtain empirical results about utility function and expected utility violation in a real although fully controlled condition, by considering decisions involving significant amount of money made under risk by ordinary decision makers.

How to reach this goal?

1) By submitting a grant proposal asking the necessary huge amount of money.

2) By observing a well controlled situation where simple but relevant decisions are taken under risk.

We have followed approach 2) by investigating choices taken by decision makers in a lottery based TV show.

Affari Tuoi

The TV show is the Italian version of the Endemol international format "Deal or no deal". In Italy it is called "Affari Tuoi".

We have watched 240 episodes of the series broadcasted during the time period from 11/2003 to 6/2005

In the Italian setting, for each episode of the show there is a single participant selected out of 20 potential participants.

Individual decisions under risk

He/she will progressively "open" a series of 20 boxes.
In each box there is indicated an amount of money.

One potential outcome is that the participant get the amount of money that remains in the last box.

Alternatively, during the game, the show director (called sometime the banker) makes some offers to the participant. He/she may be invited to

1) change his/her box;
2) ending the game accepting a specific money offer before all the remaining boxes are open.

Individual decisions under risk

The 20 prizes are the following

0.01	5000
0.20	10000
0.50	15000
1	20000
5	25000
10	50000
50	75000
100	100000
250	250000
500	500000

The expected win at the beginning is 54521.9 Euro

Individual decisions under risk

A prospect (indeed rather lucky)

The participant has still 2 boxes with 75,000 and 250,000. the banker offers her 100,000 Euro to end the show.

She answer NO to the offer.

Individual decisions under risk

Individual decisions under risk

By analyzing the 240 episodes of the show we have recorded the prospects faced and the choice taken by decision makers in the case of 738 offers.

20040131a.txt	F	A	VENETO	STUDENTE	78255.5	8000	NO	11	0.2	0.5	10	50
20040131a.txt	F	A	VENETO	STUDENTE	43845.1	16000	NO	8	0.2	0.5	10	250
20040131a.txt	F	A	VENETO	STUDENTE	70050.1	50000	SI	5	0.5	250	25000	75000
20040301a.txt	M	A	TRENTINO	VIGILEURBANO	47778.3	4000	NO	11	1	10	50	500
20040301a.txt	M	A	TRENTINO	VIGILEURBANO	39100.2	10000	NO	5	1	500	20000	75000
20040302a.txt	F	B	TOSCANA	AUTOTRASPORTATORE	10469.7	1000	NO	11	0.5	1	5	10
20040302a.txt	F	B	TOSCANA	AUTOTRASPORTATORE	10020.8	3000	NO	8	1	5	10	50
20040302a.txt	F	B	TOSCANA	AUTOTRASPORTATORE	15011.2	6000	NO	5	1	5	50	25000
20040303a.txt	M	C	MOLISE	AUTORIPARATORE	35914.2	3000	NO	11	0.01	0.2	0.5	1
20040304a.txt	F	B	VENETO	INFERMIERA	11400.9	1000	NO	11	0.01	10	50	100
20040304a.txt	F	B	VENETO	INFERMIERA	8367.78	2000	NO	9	0.01	10	50	250
20040304a.txt	F	B	VENETO	INFERMIERA	14002	4000	NO	5	10	10000	15000	20000
20040305a.txt	F	A	UMBRIA	NA	35491.5	3500	NO	11	1	5	50	100
20040305a.txt	F	A	UMBRIA	NA	9080	3500	NO	5	50	100	250	20000
20040305a.txt	F	A	UMBRIA	NA	10025	10000	SI	2	50	20000	0	0

Here I provide some descriptive statistics:

Gender of participants:

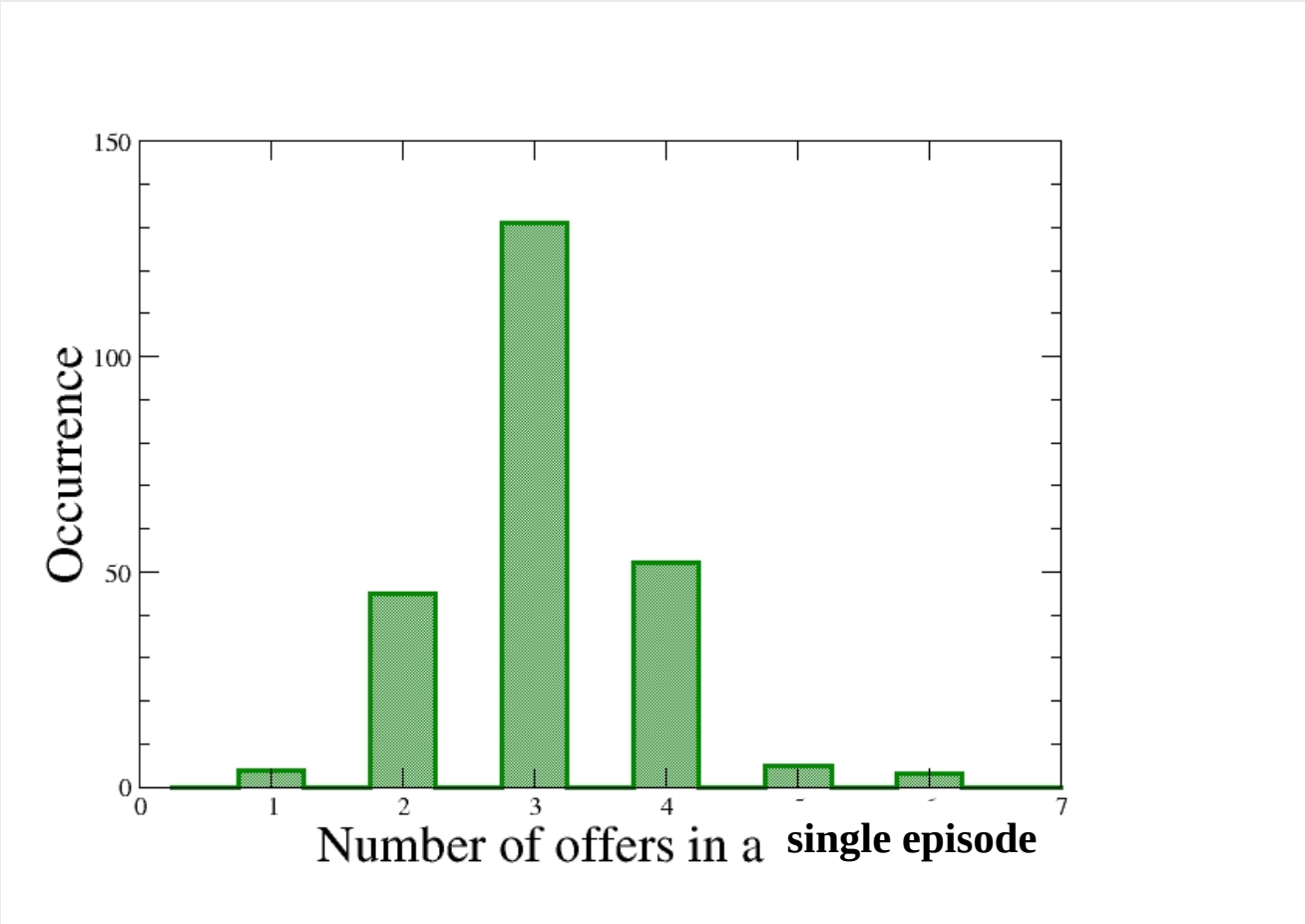
Females	119
Males	121

Amount of money offered

Total	12,189,737.5	Euro
minimum	2.5	Euro
maximum	200,000	Euro
average	16,517.2	Euro

Individual decisions under risk

The number of offers in a single episode is not fixed. It is ranging from 1 to 6.



On 117 cases over 240 an offer has been accepted ending the game

	Accepting offers	All participants
Females 54	A 15 B 16 C 19 D 4	A 30 B 42 C 34 D 13
Males 63	A 13 B 18 C 11 D 21	A 20 B 38 C 28 D 35

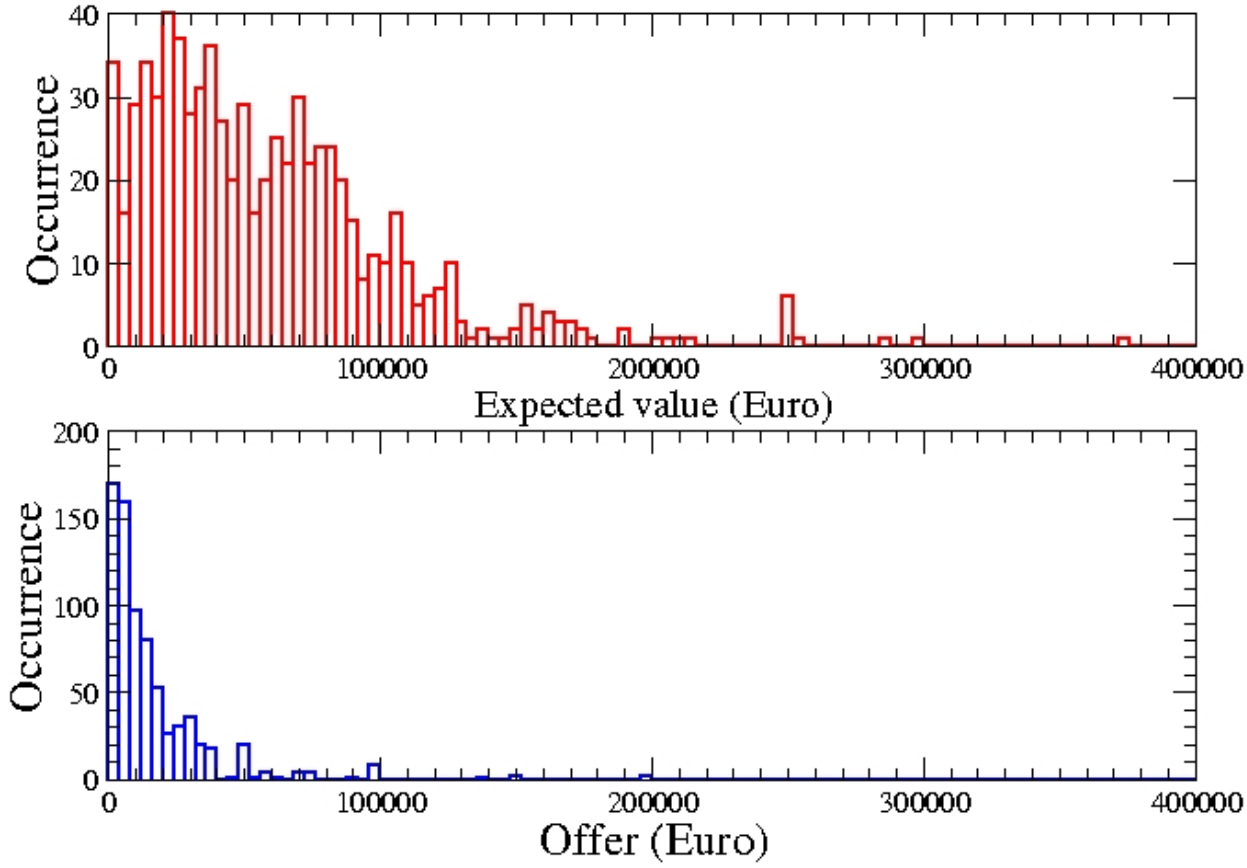
The amount of money of accepted offers

Total	3,891,770	Euro
minimum	110	Euro
maximum	200,000	Euro
average	33,262.99	Euro

is less than the value of expected win at the moment of the offer indicating risk aversion

Total	8,579,954	Euro
minimum	113.1	Euro
maximum	375,000	Euro
average	73,332.94	Euro

Individual decisions under risk



Here we attempt to provide a more quantitative evaluation of the degree and profile of risk aversion by characterizing the utility function of decision makers

We assume a utility function of the form

$$U(x) = cx^{\gamma}$$

When a decision maker refuses an offer x_{off} under a prospect x_i $i=1,\ldots,k$ of remaining potential prizes, expected utility theory implies the inequality

$$U(x_{off}) < \frac{1}{k}\sum_{i=1}^{k} U(x_i) \qquad (1)$$

on the contrary, in case of acceptance one has

$$U(x_{off}) > \frac{1}{k}\sum_{i=1}^{k} U(x_i) \qquad (2)$$

Inequality (1) provides a lower limit of the exponent γ_{low}

This value is obtained by solving the equation

$$x_{off}{}^{\gamma_{low}} = \frac{1}{k}\sum_{i=1}^{k} x_i{}^{\gamma_{low}}$$

whereas inequality (2) provides an upper limit of the exponent γ_{up} by solving the same equation

$$x_{off}{}^{\gamma_{up}} = \frac{1}{k}\sum_{i=1}^{k} x_i{}^{\gamma_{up}}$$

Episode of 14 Nov 2003

Offer number 3 of 21000 Euro

Prizes still available are 0.01, 10, culla=250, 75000 and 500000

Episode of 14 Nov 2003

Offer number 3 of 21000 Euro

The decision of the participant is NO implying

$$U(21000) < \frac{1}{5}\left[U(0.01) + U(10) + U(250) + U(75000) + U(500000)\right]$$

Under the assumption of pore law utility function we estimate

$$\gamma > \gamma_{low(3)} = 0.37$$

It is worth noting that the expected win at this stage is 115052 Euro

Episode of 14 Nov 2003

Offer number 1 and 2 of 3500 and 7000 Euro were also previously refused

These negative decisions are also implying

$$\gamma > \gamma_{low(1)} = 0.26$$

$$\gamma > \gamma_{low(2)} = 0.32$$

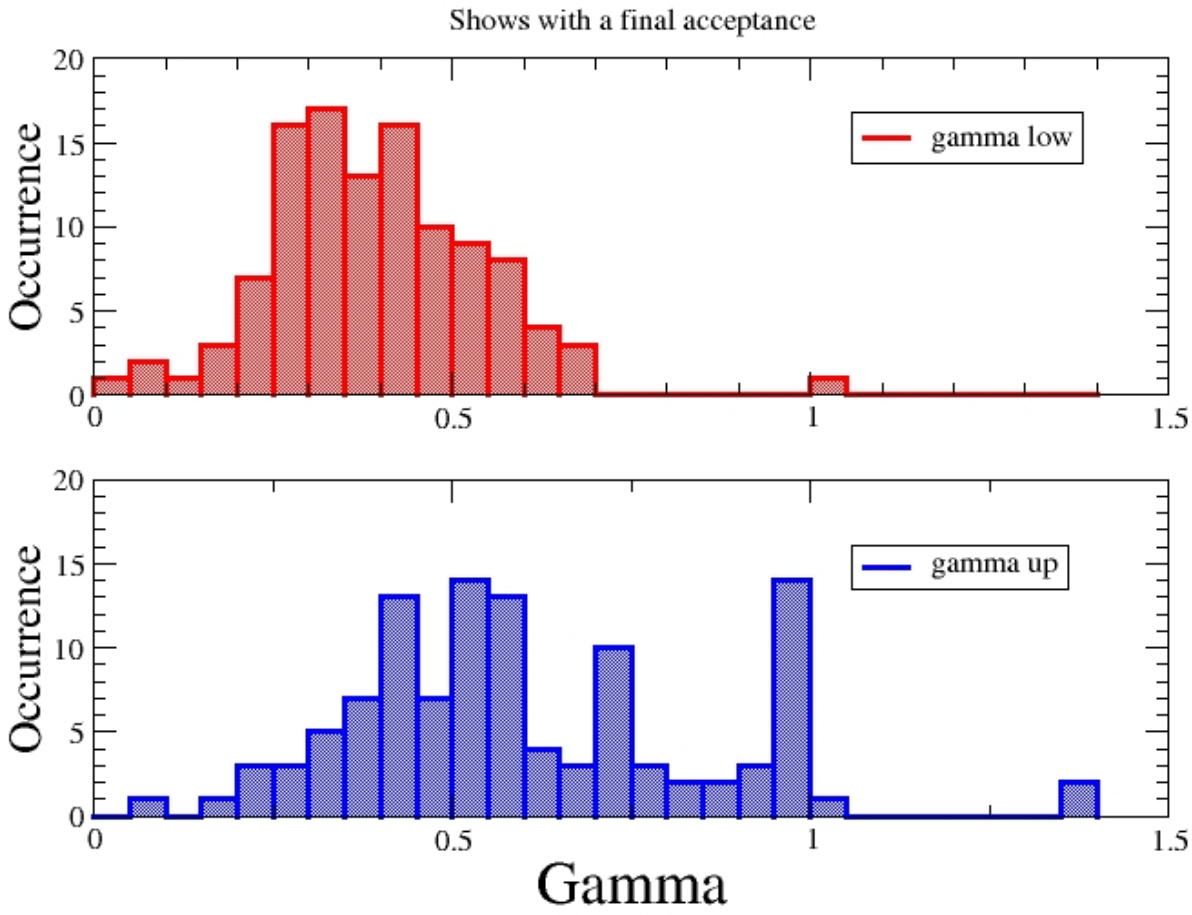
Therefore for this participant

$$\max\{\gamma_{low}\} = 0.37$$

For each decision maker we have estimated $\max\{\gamma_{low}\}$ and, when available, γ_{up}

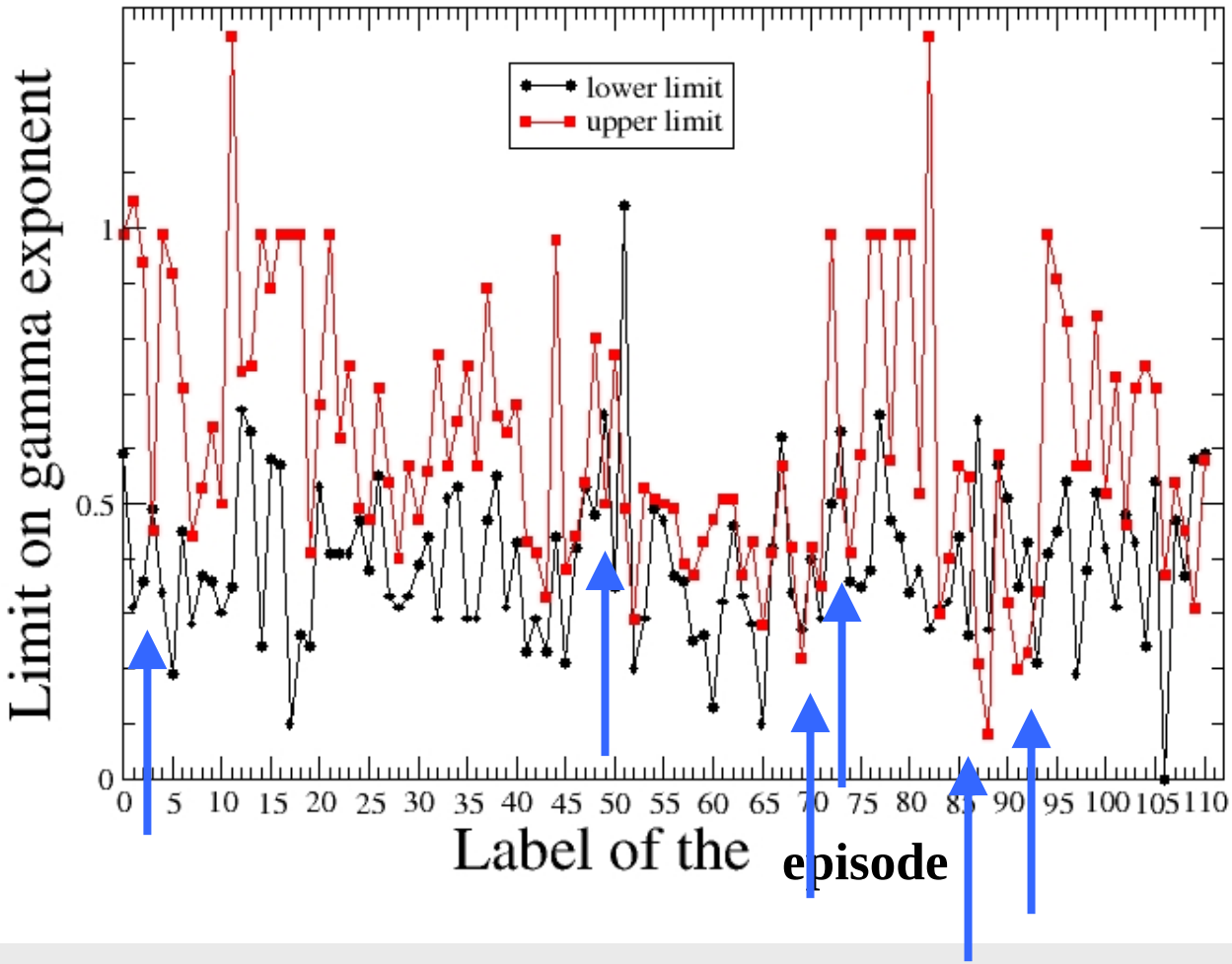
	#	mean	std	min	max
$\max\{\gamma_{low}\}$	238	0.46	0.20	0.10	1.04
γ_{up}	112	0.61	0.25	0.08	1.35

Their pdfs are



Violation of expected utility theory

Episodes with accepted orders



One way to see the violation is to detect the crossing of the two curves.

Conclusions

- An utility function of the functional form $U(x) = cx^{\gamma}$ is compatible with decision taken under risk for levels of real money as large as 200,000 Euro

- The value of the exponent γ is within the interval $0.46 < \gamma < 0.61$

- Violation of expected utility theory are detected in experiment involving a significant amount of real money.

The OCS group

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