SOME STYLIZED FACTS REVISITED: CONSEQUENCES FOR FLUCTUATION SCALING

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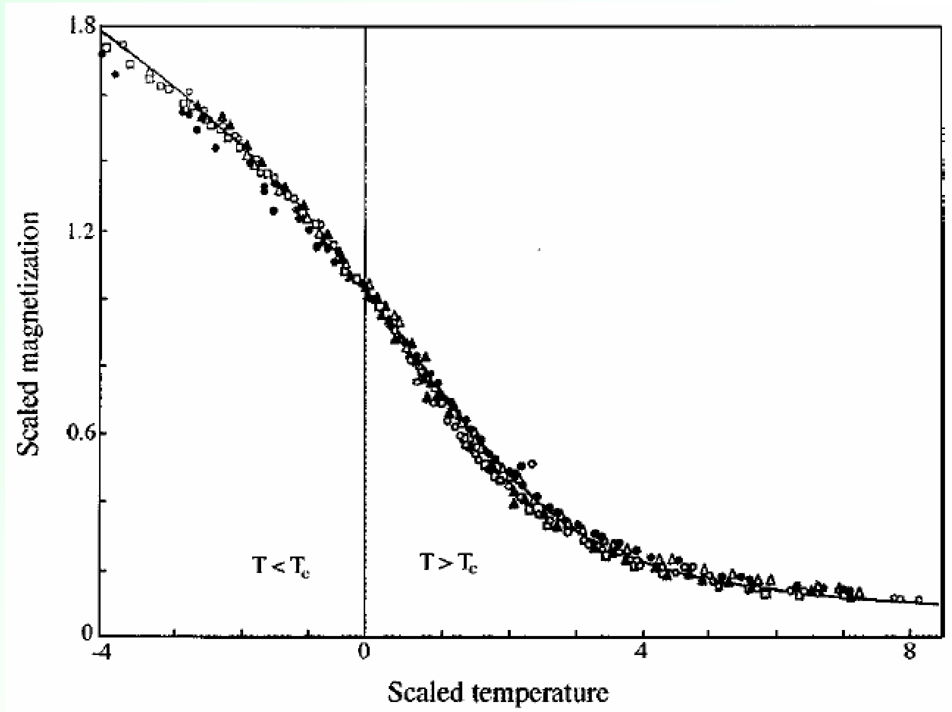
Outline

- Universality in Physics (!) and Finance (?)
- Fluctuation scaling – universality classes
- Fluctuation (multi-)scaling in finance
- Volume scaling revisited
- Size matters: Non-universal scaling
- Conclusions

Universality in physics

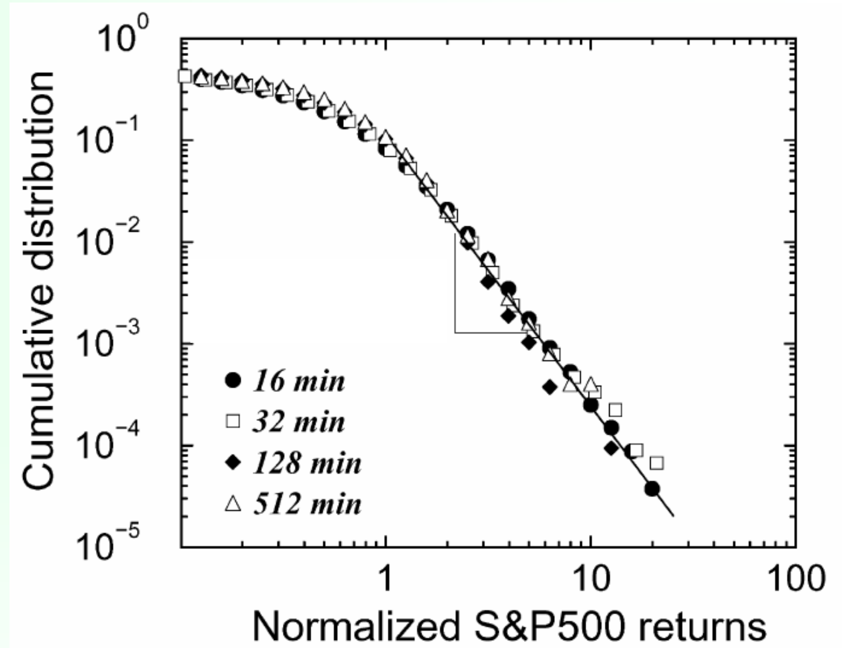
- 5 different magnetic materials ($CrBr_3$, EuO, Ni, YIG, Pd_3Fe)
- the curves collapse



- "different systems behave the same"
- power law behavior, e.g., $M \propto H^{1/\delta}$

Universality in finance

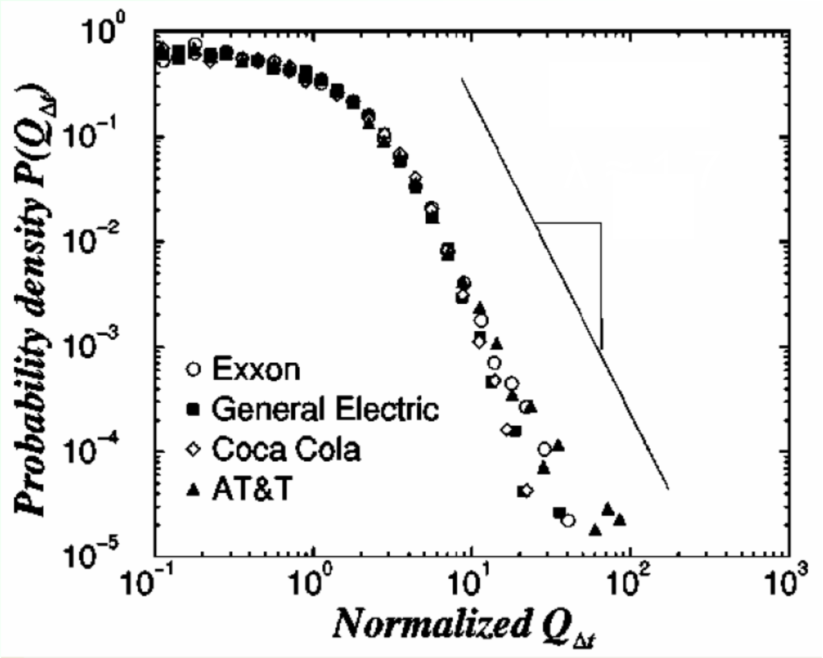
- the "inverse cube law"
  - price changes (returns)
  - 1000 NYSE companies
  - outside Levy regime



P. Gopikrishnan et al., Physica A 287, 362-373 (2000)

Universality in finance?

- the "inverse half cube law"
  - trading volume (*Q*) or value (*f*)
  - Levy stable regime



Delicate questions related to extrapolation

P. Gopikrishnan et al., Phys. Rev. E 62, 4493 (2000)

Fluctuation scaling

Activity at site *i* in a multi-channel observation: $f_i(t)$

E.g. traffic at induction loops, web-site visits, packages through a router, current at a circ. element...

$$\langle f_i \rangle = \frac{1}{T}\sum_{t=0}^{T} f_i(t)$$

$$\sigma_i = \sqrt{\langle f_i(t)^2 \rangle - \langle f_i(t) \rangle^2}$$

Fluctuation scaling: $\sigma_i \sim \langle f_i(t) \rangle^{\alpha}$

Found in many systems

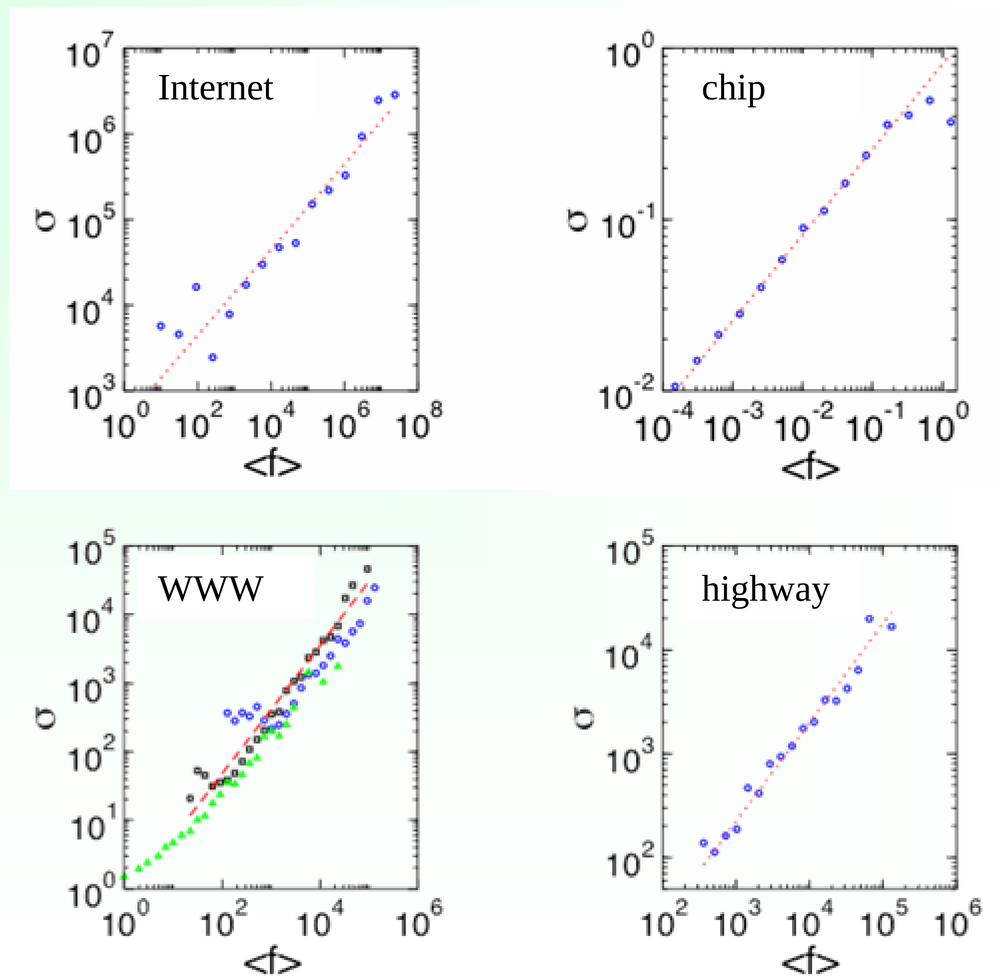
Menezes, Barabási 2004

Universality classes in fluctuation scaling

$\alpha = 1/2$ →

$$\sigma_i \sim \langle f_i(t) \rangle^{\alpha}$$

$\alpha = 1$ →



internally driven

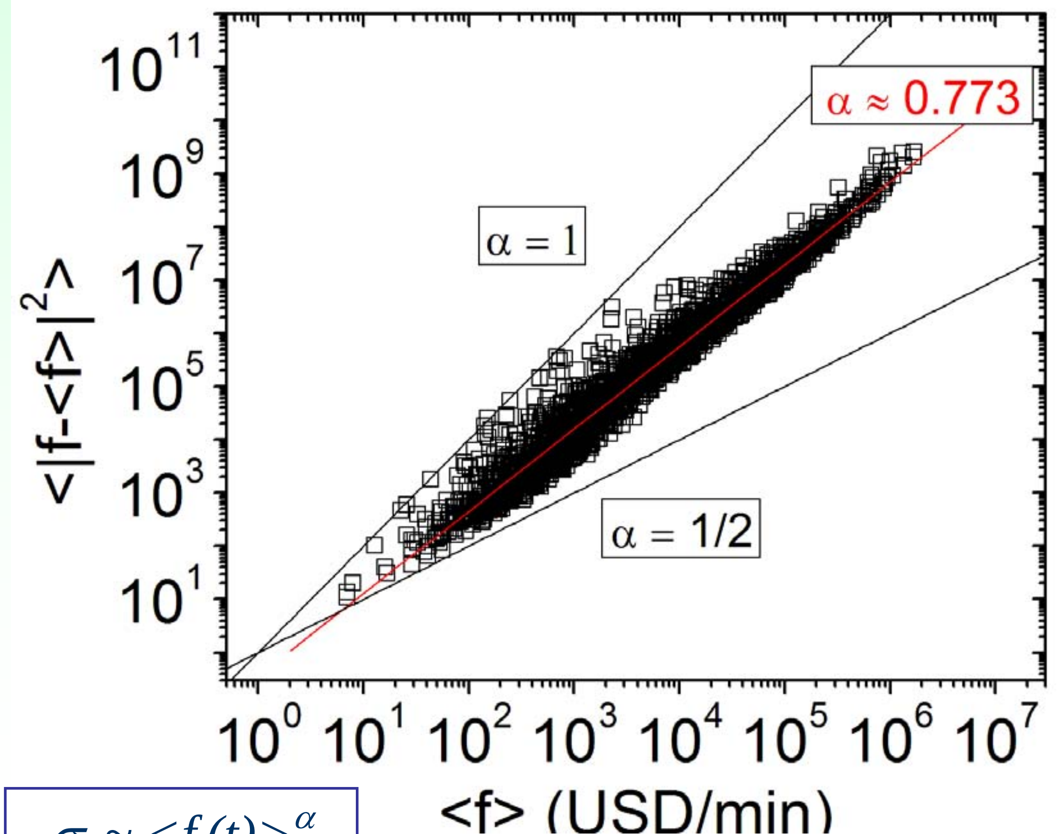
externally

Menezes, Barabási 2004

Fluctuation scaling in stock market data

Activity: flow

$$f_i^{\Delta t}(t) = \sum_{\tau \in [t, t+\Delta t]} Y_i(\tau) V_i(\tau)$$



Y_i price of stock i,

V_i is traded volume

parameter:

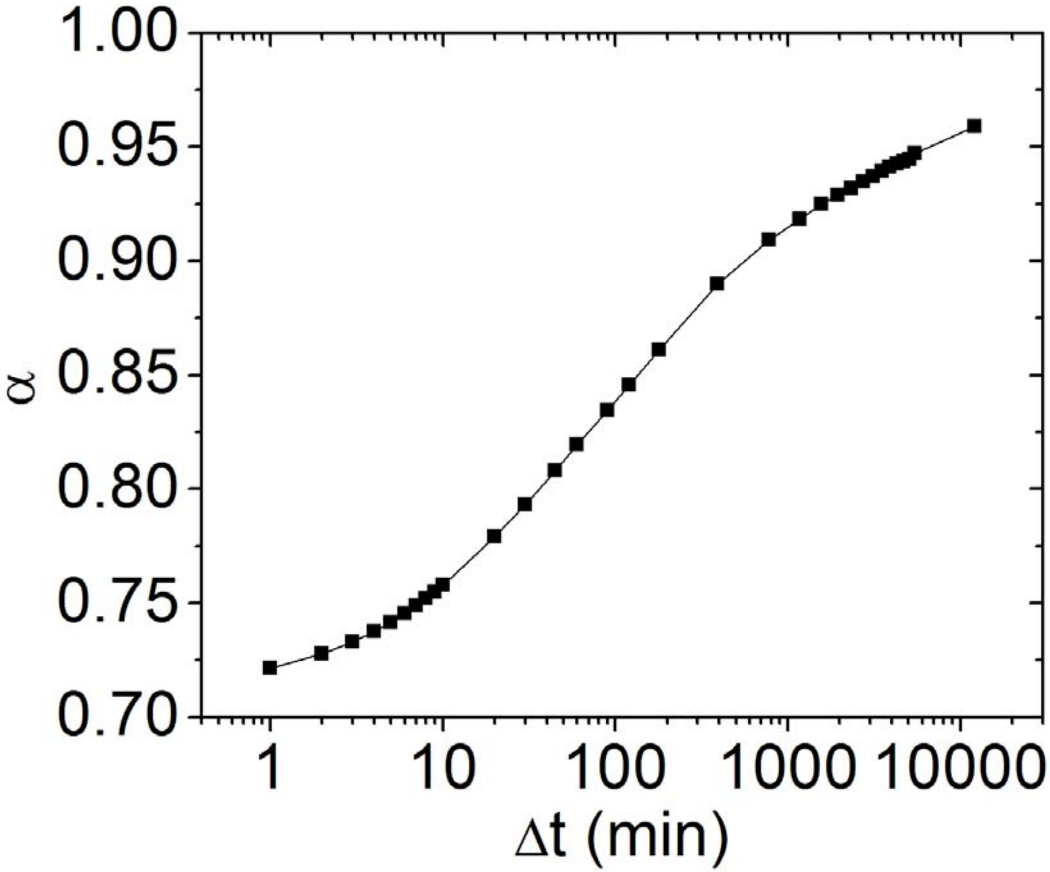
time difference Δt

Here: Δt=10min

$$\sigma_i \sim <f_i(t)>^{\alpha}$$

Eisler et al., 2005

Results for stock market data



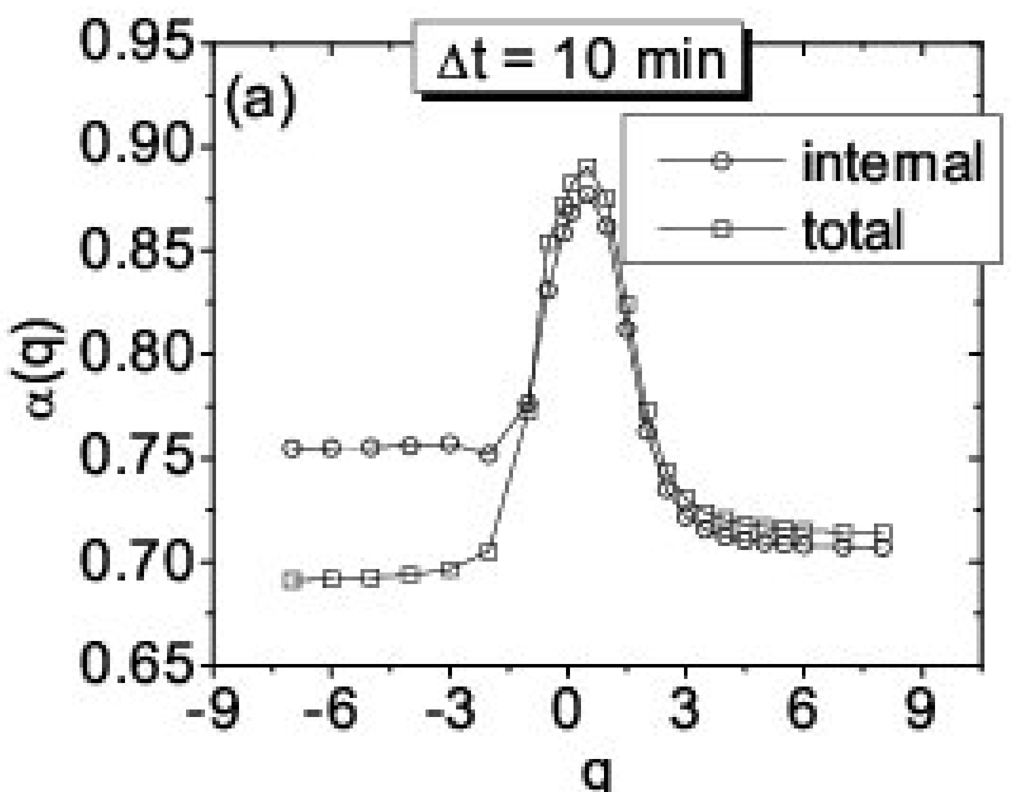
Δt-dependent, non-universal exponents

$\sigma_i \sim \langle f_i(t) \rangle^{\alpha}$

Eisler et al., 2005

Multiscaling in stock market data

$$\left\langle \left| f_i - \langle f_i \rangle \right|^q \right\rangle = C_f^q(q; \quad) \langle f_i \rangle^{q\alpha(q; \quad)}$$

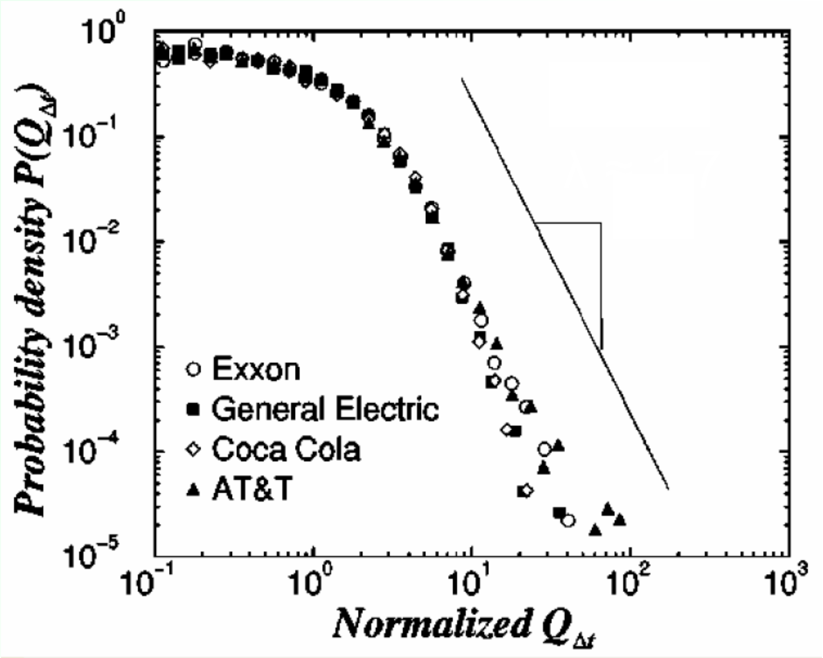


- Nontrivial q-dependence of the scaling exponent
- Not observed in other systems

Eisler et al., 2005

The distribution of traded value

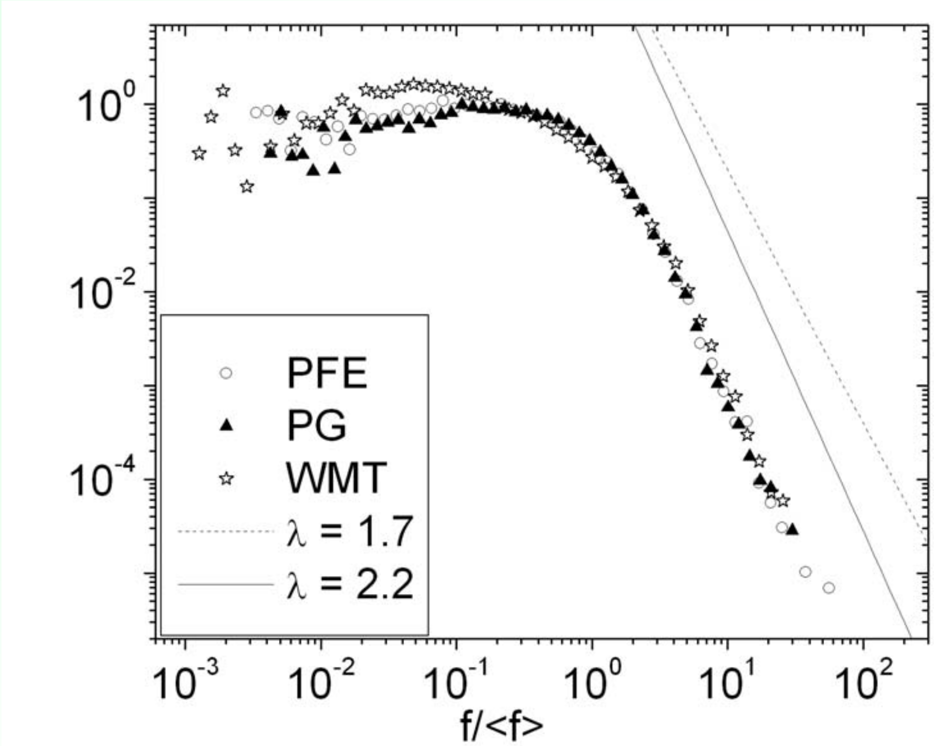
- the "inverse cube half law"
  - trading volume (*Q*) or value (*f*)
  - Levy stable regime: No second moment!



P. Gopikrishnan et al., Phys. Rev. E 62, 4493 (2000)

The distribution of traded value

- analysis of 1000 top companies using extensions of Hill's method
- tail exponent greater than 2
  - Gaussian stable regime



Z. Eisler, J. Kertész, arXiv:physics/0508156

Beyond averages: The distribution of traded value

- fat tails
- tail exponent greater than 2
- analysis of 1000 top companies
- increasing effective exponents
  - Gaussian stable regime

Δt	λ
1 min	2.4 ± 0.23
5 min	2.8 ± 0.5
15 min	3.1 ± 0.6
60 min	3.45 ± 0.8
120 min	3.8 ± 1.1
390 min	5.1 ± 0.8

Z. Eisler, J. Kertész, arXiv:physics/0508156

Non-universal correlations of traded value (1)

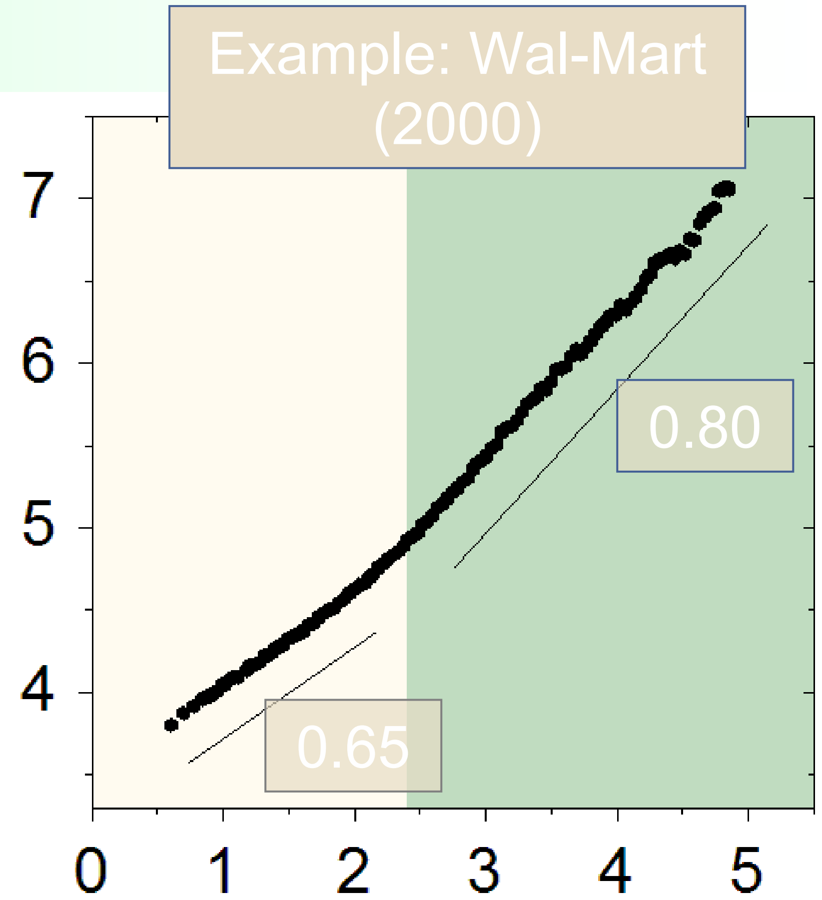
- The measured tail exponents are larger than 2
  - standard deviation exists on all time scales
  - for a stock *i*, the Hurst exponent *H(i)* can be defined as

$$\sigma_i(\Delta t) = \left\langle \left(f_{\Delta t} - \langle f_{\Delta t} \rangle \right)^2 \right\rangle = C_i \Delta t^{H(i)}$$

- persistent: H > 0.5
- uncorrelated: H = 0.5

Non-universal correlations of traded value (2)

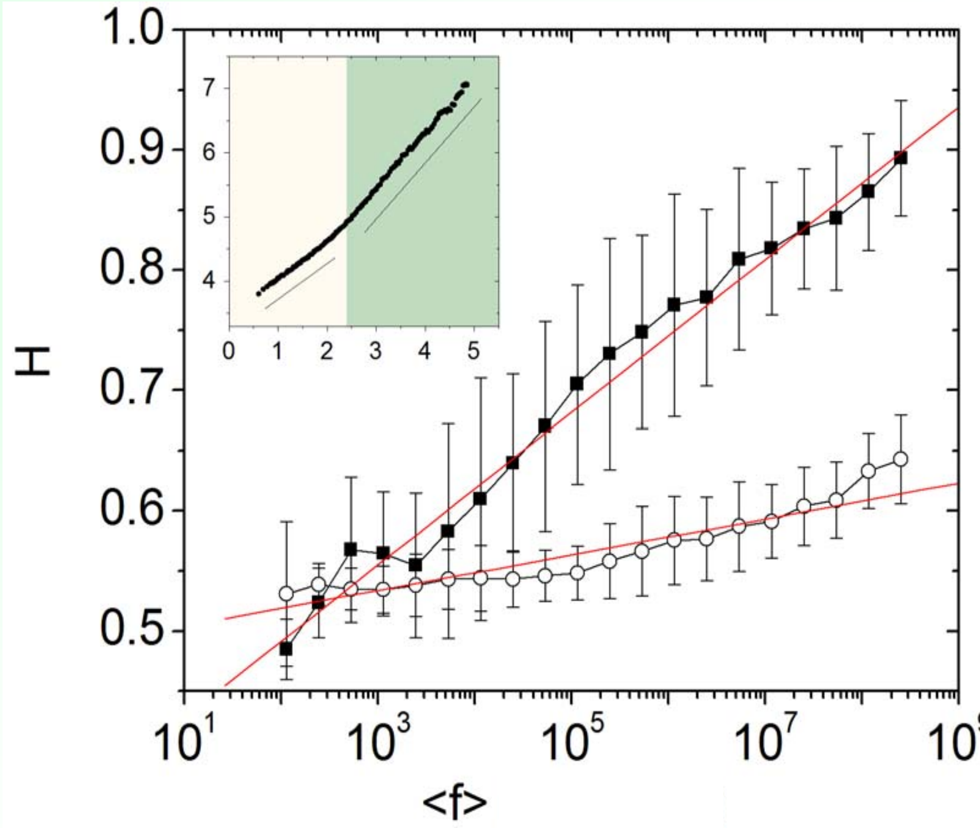
- Stocks display a crossover
  - at $\Delta t \approx 390$ min = 1 day
  - from weaker to stronger correlation



Z. Eisler, J. Kertész, arXiv:physics/0508156

Capitalization dependence

- H non-universal!
- depends on $\langle f \rangle$ which is a monotonous function of the company size
  - γ(Δt < 250 min) = 0.016 ± 0.001
  - γ(Δt > 630 min) = 0.063 ± 0.002



$$H(\langle f \rangle) = H(\langle f \rangle = 1) + \gamma \log \langle f \rangle$$

Z. Eisler, J. Kertész, arXiv:physics/0508156

Non-universal correlations of traded value (3)

- $\langle f \rangle$ strongly depends on capitalization
  - capitalization acts as a parameter that determines the strength of correlations present in trading activity
  - the effect is weak on an intraday scale
  - it is much stronger for day-to-day fluctuations
- a clear logarithmic law: only the order of magnitude matters!

$$H(\langle f \rangle) = H(\langle f \rangle = 1) + \gamma \log \langle f \rangle$$

Z. Eisler, J. Kertész, arXiv:physics/0508156

Relation to fluctuation scaling

$$\sigma_i(\Delta t) = \left\langle \left(f_{\Delta t} - \langle f_{\Delta t} \rangle \right)^2 \right\rangle = C_i \Delta t^{H(i)}$$

$$\sigma_i \sim \langle f_i(t) \rangle^{\alpha}$$

- A self-consistent scheme
- γ the same for both scalings
- Multiscaling in Δt too

For details see

Eisler and Kertész: PRE & EJP (in press) + archive

Summary

- Scaling and universality concepts have to be handled with care in finance
- No inverse cube half law -> 2-nd moment of volume distribution exists
- Size matters! log dependence of H
- Fluctuation (multi-)scaling observed
- Related to volume (multi-)scaling

THANK YOU