„Multifractality and self–similarity in the Baltic States market”

Igoris Belovas, <u>Audrius Kabašinskas</u>, Leonidas Sakalauskas

Institute of Mathematics and Informatics, Operational Research Sector at Data Analysis Department

Multifractality and self-similarity

Hypothesis of fractionallity or self–similarity arises due to that financial series are not properly described by Gaussian models.

The Hurst exponent is used to characterize fractionallity.

Multifractality and self-similarity

The process with the Hurst index H=½ corresponds to the Brownian motion when variance increases at the rate of $\sqrt{t}$, where t is the amount of time. Indeed, in real data this growth rate (Hurst exponent) is longer t^H

Hurst index

There are many methods to evaluate Hurst index but in literature the following are usually used:

- Time-domain estimators (*Absolute Value method(Absolute Moments),Variance method (Aggregate Variance),* ***R/S method****, Variance of Residuals*);
- Frequency-domain/wavelet-domain estimators (*Periodogram method ,Whittle,Abry-Veitch (AV)*).

Multifractality and self-similarity

As $0.5<H\leq1$, the Hurst exponent implies a persistent time series characterized by long memory effects, and when $0\leq H<0.5$, it implies an anti-persistent time series that covers less distance than a random process.

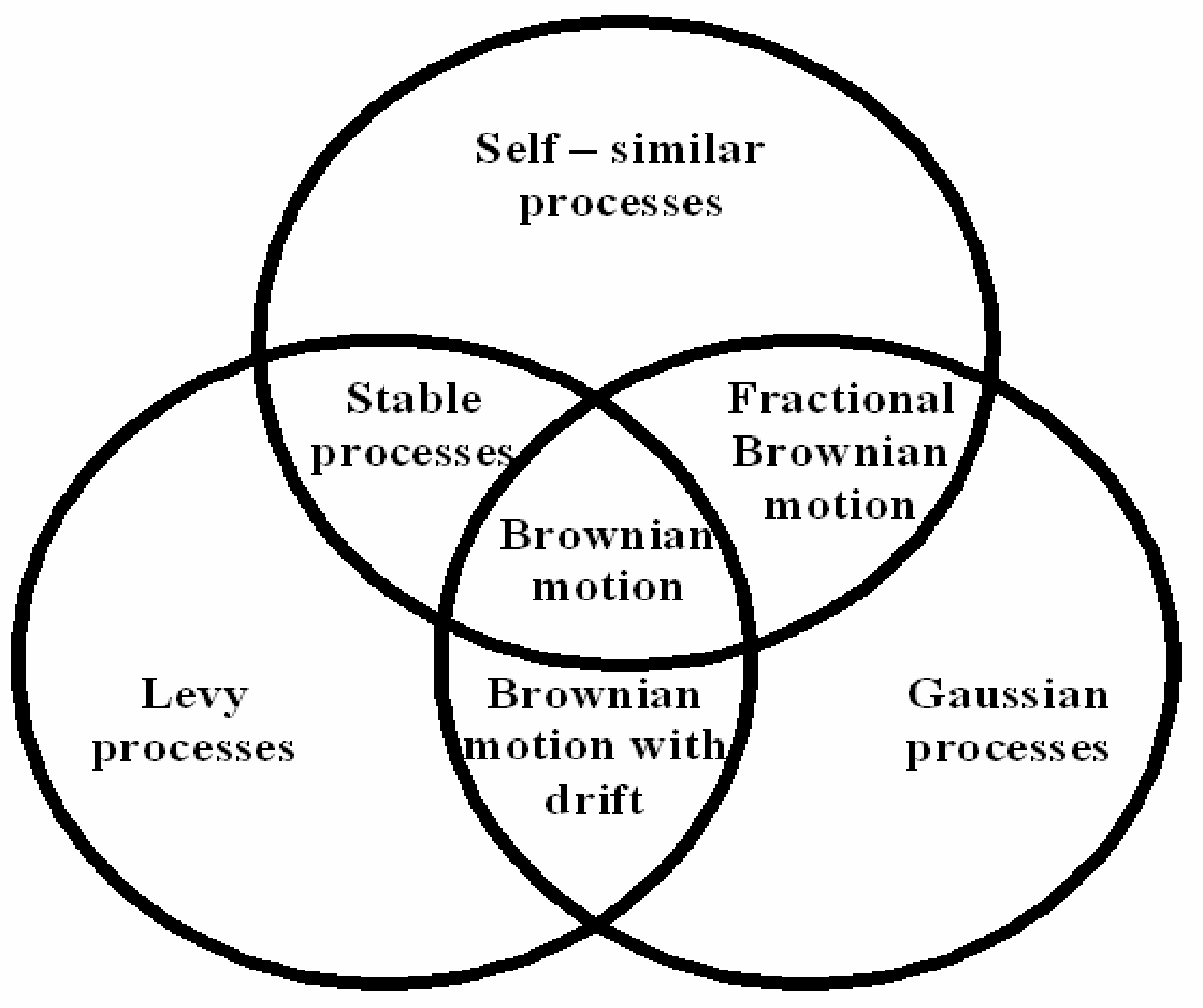
Self-similarity

Continuous time process $Y = \{Y(t), t \in T\}$ is self-similar, with the self-similarity parameter H (Hurst index), if it satisfies the condition:

$$Y(t) \overset{d}{=} a^{-H} Y(at), \quad \forall t \in T, \forall a > 0, 0 \leq H < 1$$

where the equality is in the sense of finite-dimensional distributions.

Self-similar processes and their relation to Levy processes



Self-similar processes and their relation to Levy processes

Suppose a Levy process $X=\{X(t), t\geq 0\}$. Then X is self-similar if and only if each $X(t)$ is strictly stable. The index α of stability and the exponent H of self-similarity satisfy

$$\alpha = 1/H$$

α-stable distribution

We say that a r.v. X is distributed by the stable law and denote

$$X \sim S_\alpha(\sigma, \beta, \mu)$$

where S_α is the probability density function.

Each stable distribution $S_\alpha(\sigma,\beta,\mu)$ has the stability index $\alpha \in (0;2]$, which can be treated as the main parameter, when we make investment decision, $\beta \in [-1,1]$ is the parameter of asymmetry, $\sigma > 0$ is that of scale, $\mu \in \boldsymbol{R}$ is the parameter of position.

Definition of α-stable distribution

A random variable r is said to have a stable distribution if for any $n > 2$, there is a positive number C_n and a real number D_n such that

$$r_1 + r_2 + ... + r_n \sim C_n r + D_n$$

where $r_1, r_2, ..., r_n$ are independent copies of r.

If D_n=0 we have strictly stable r.v.

Special cases of α-stable distribution

- The Gaussian distribution, if α=2, β=0 ;
- The Cauchy distribution, if α=1, β=0;
- The Levy distribution, if α=1/2, β=0, whose density

$$p(x) = \begin{cases} \frac{1}{\sqrt{2\pi x^3}} e^{-x/2} & x > 0 \\ 0 & x \le 0 \end{cases}$$

- Degenerate distribution, if α=0, σ=0.

Stable processes

A stochastic process $\{X(t), t \in T\}$ is strictly stable if all its finite dimensional distributions are strictly stable.

Theorem. Let $\{X(t), t \in T\}$ be a stochastic process. $\{X(t), t \in T\}$ is strictly stable if and only if all linear combinations

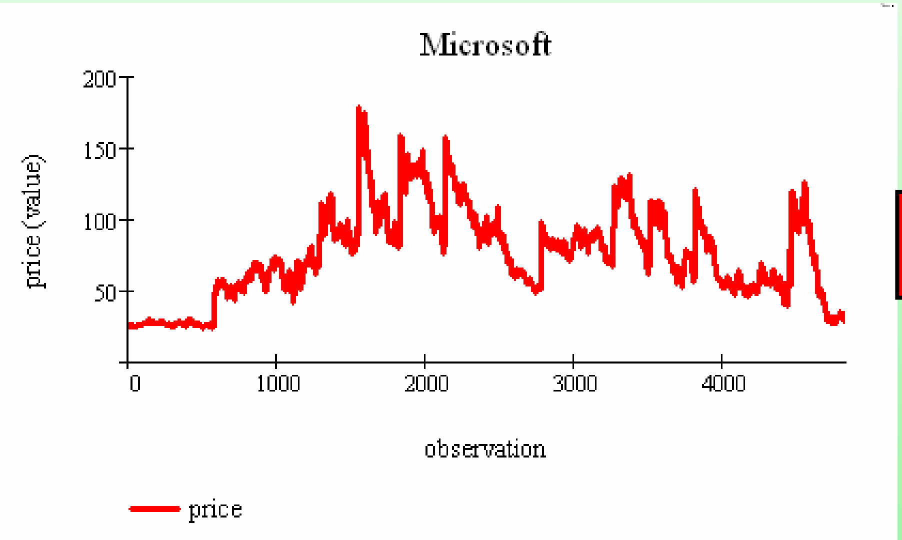
$$\sum_{k=1}^{d} b_k X(t_k)$$

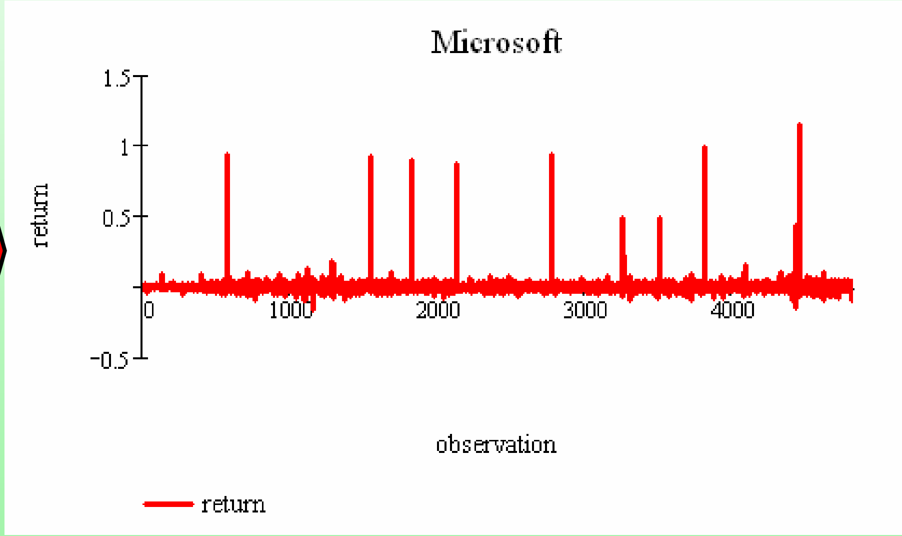
(here $d \geq 1$ $b_1, b_2, \ldots, b_d$ – real) are strictly stable.

Data

P_i – observed stock price

$$X_i = \frac{P_{i+1} - P_i}{P_i}$$ - return





Problems

The Baltic States market is comparatively new (series are short 1000 – 2000 data points) and passive (number of daily transactions is comparatively small).

Daily stock return is a continuous random variable with some distribution function (Gaussian, stable etc.). But in real market, when stock price does not change, its return is equal to 0. When number of such observations increases, then variance tends to 0 and the distribution function becomes degenerate distribution function.

Daily return problem
(application for the Baltic States equity)

We analyzed all the Baltic Main list and Baltic I-list in period 2000 – 2006. Number of daily zero stock returns for this period differs from 12% to 89% and in average it is 52% !! Any distribution function not fitted to the empirical data (Anderson–Darling and Kolmogorov–Smirnov goodness–of–fit tests).

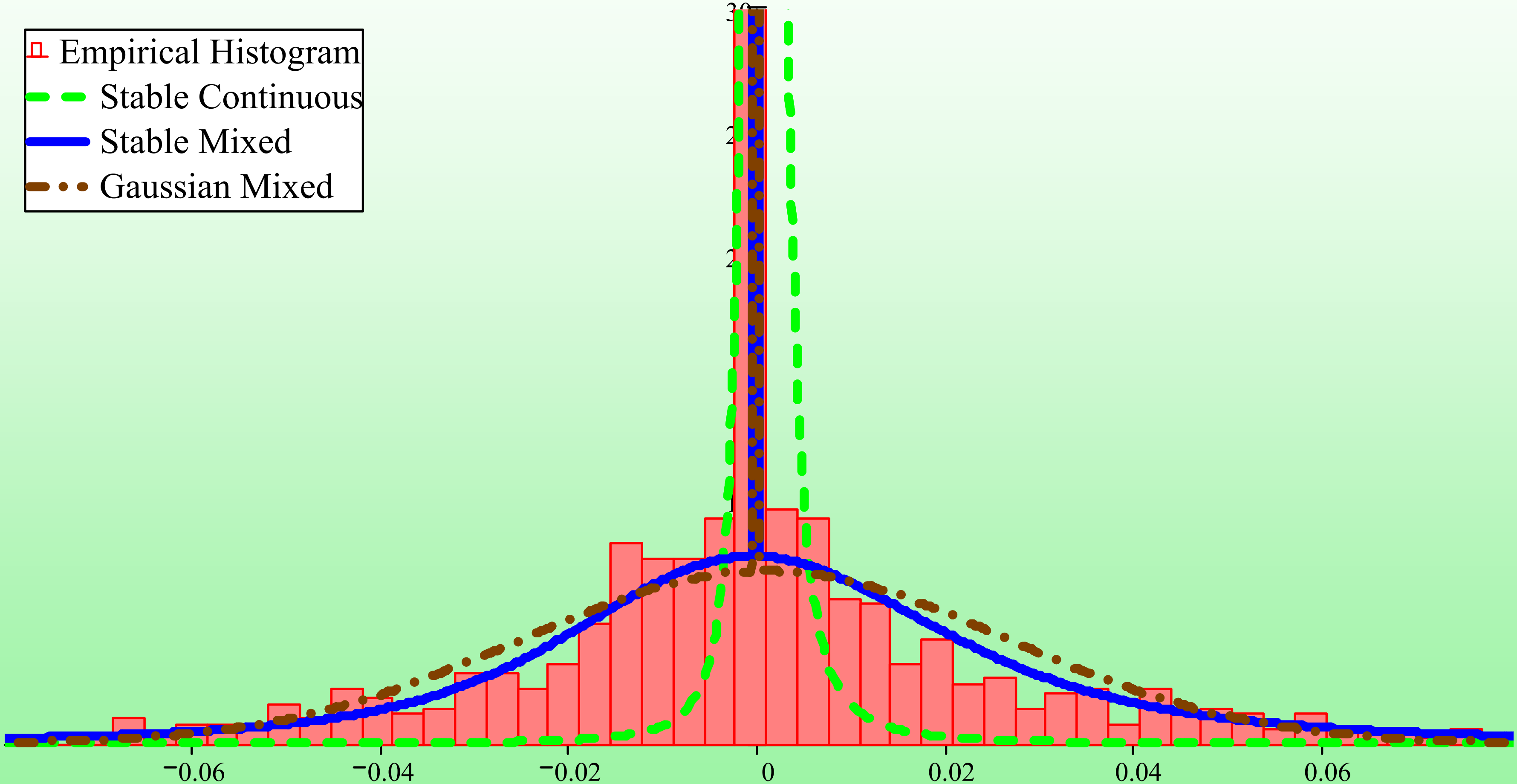
Daily return problem

(solution)

So, we removed zero values from series (probability P(X=0)=0 in continuous case) and analyzed "non–zero" series.
There was found 49 stable series (6 of them were Gaussian) in other 16 cases any distribution function did not fitted to the empirical data.

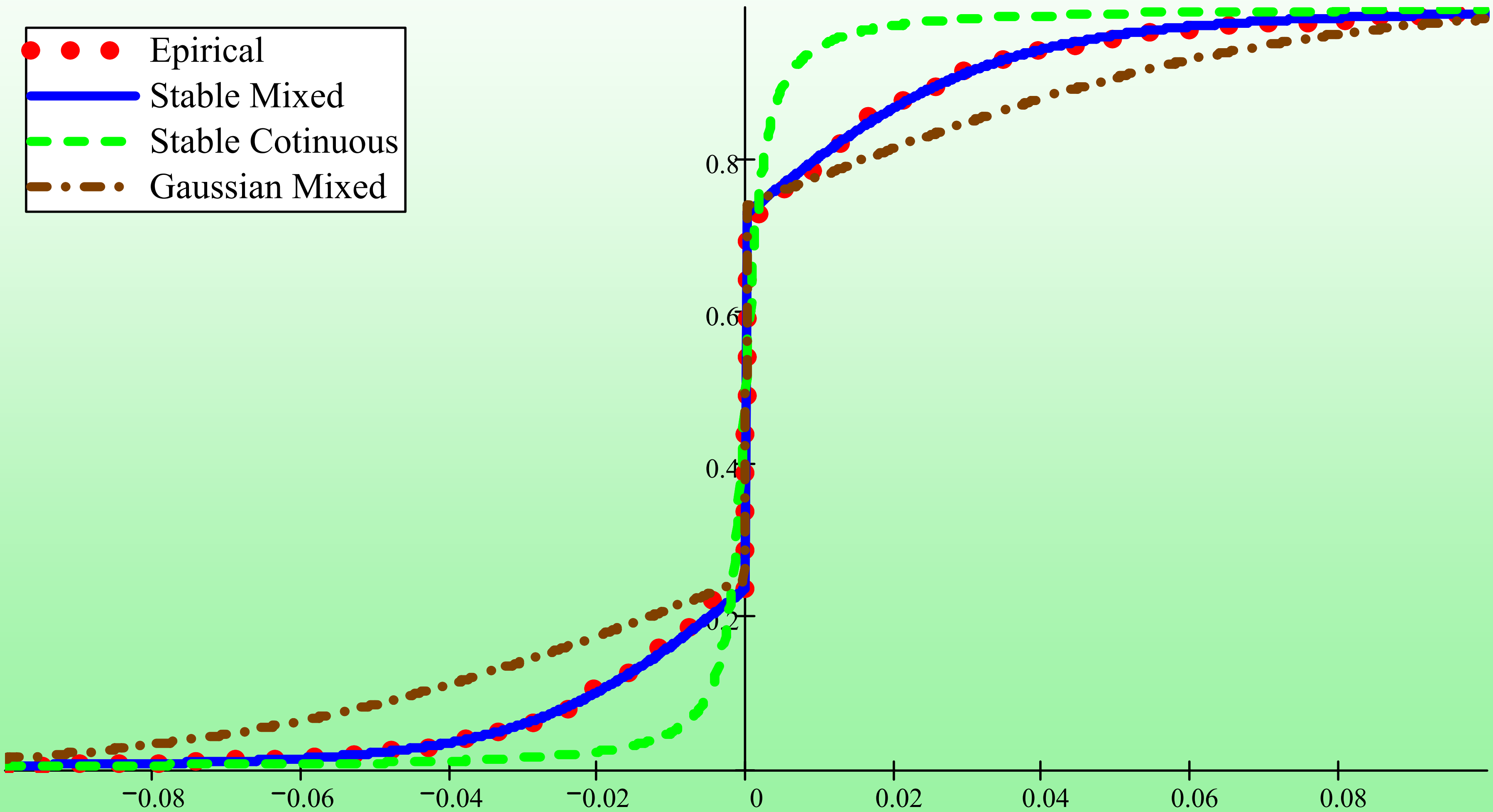
Daily return problem

(mixed pdf's and histogram)

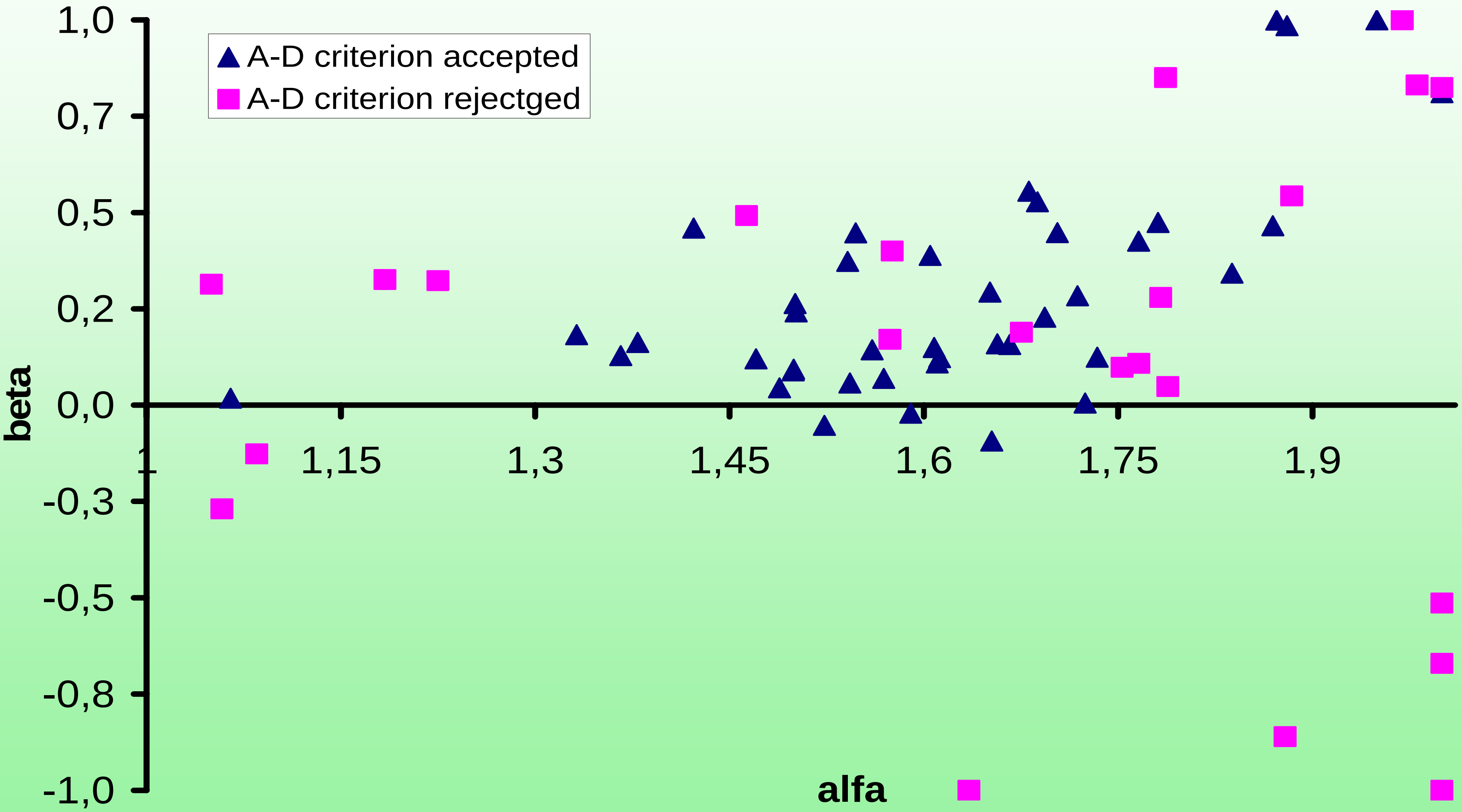


Daily return problem

(Empirical cdf and mixed cdf's)



Stability analysis



Multifractality and self-similarity

Self-similarity is often investigated through the behavior of the absolute moments.

Consider the aggregated series $X(m)$, obtained by dividing a given series of length N into blocks of length m, and averaging the series over each block

$$X^{(m)}(k) = \frac{1}{m} \sum_{i=(k-1)m+1}^{km} X_i \qquad \text{, where } k=1, 2\ldots[N/m].$$

Consider

$$AM^{(m)}(q) = E\left|\frac{1}{m}\sum_{i=1}^{m} X(i)\right|^{q} = \frac{1}{m}\sum_{k=1}^{m}\left|X^{(m)}(k) - \overline{X}\right|^{q}$$

Multifractality and self-similarity

If X is self-similar, then $AM^{(m)}(q)$ is proportional to $m^{b(q)}$, it means that $\ln AM^{(m)}(q)$ is linear in $\ln m$ for a fixed q:

$$\ln AM^{(m)}(q) = \beta(q) \ln m + C(q)$$

In addition, the exponent $\beta(q)$ is linear with respect to q. In fact, since

$$X^{(m)}(i) \overset{d}{=} m^{1-H} X(i)$$

Multifractality and self-similarity

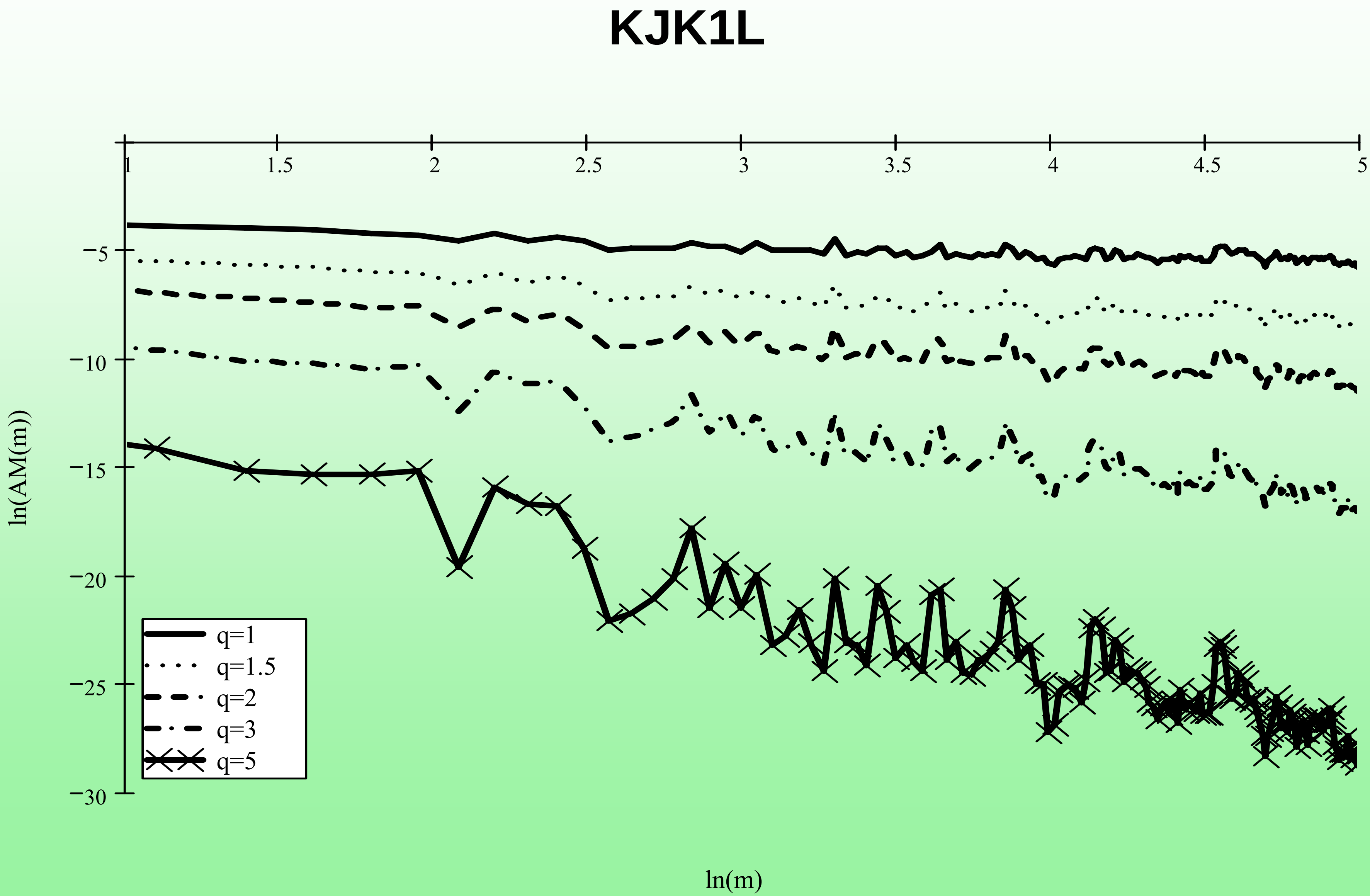
we have

$$\beta(q) = q(H-1)$$

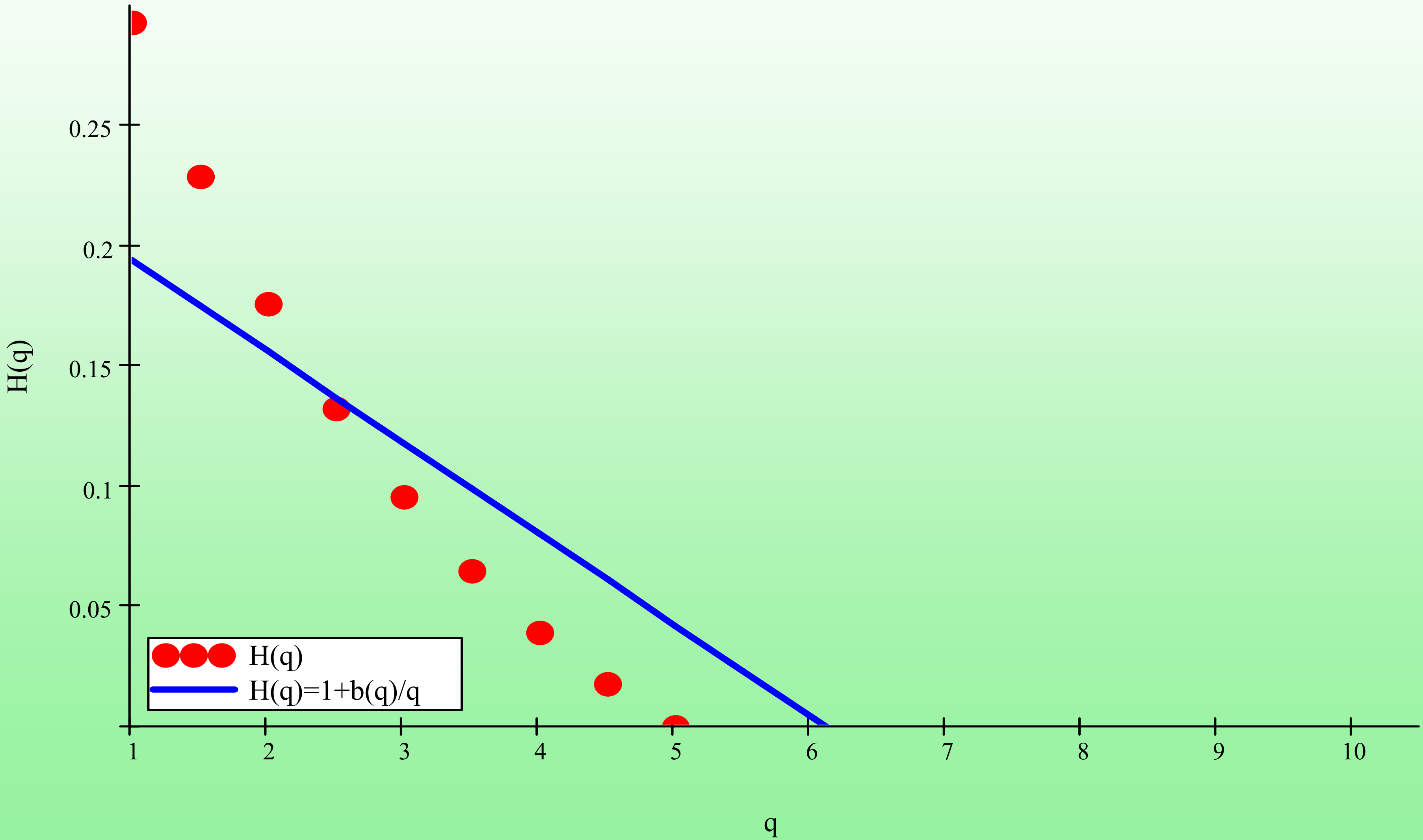
is linear on q .

If here is no linearity a process is only multifractal.

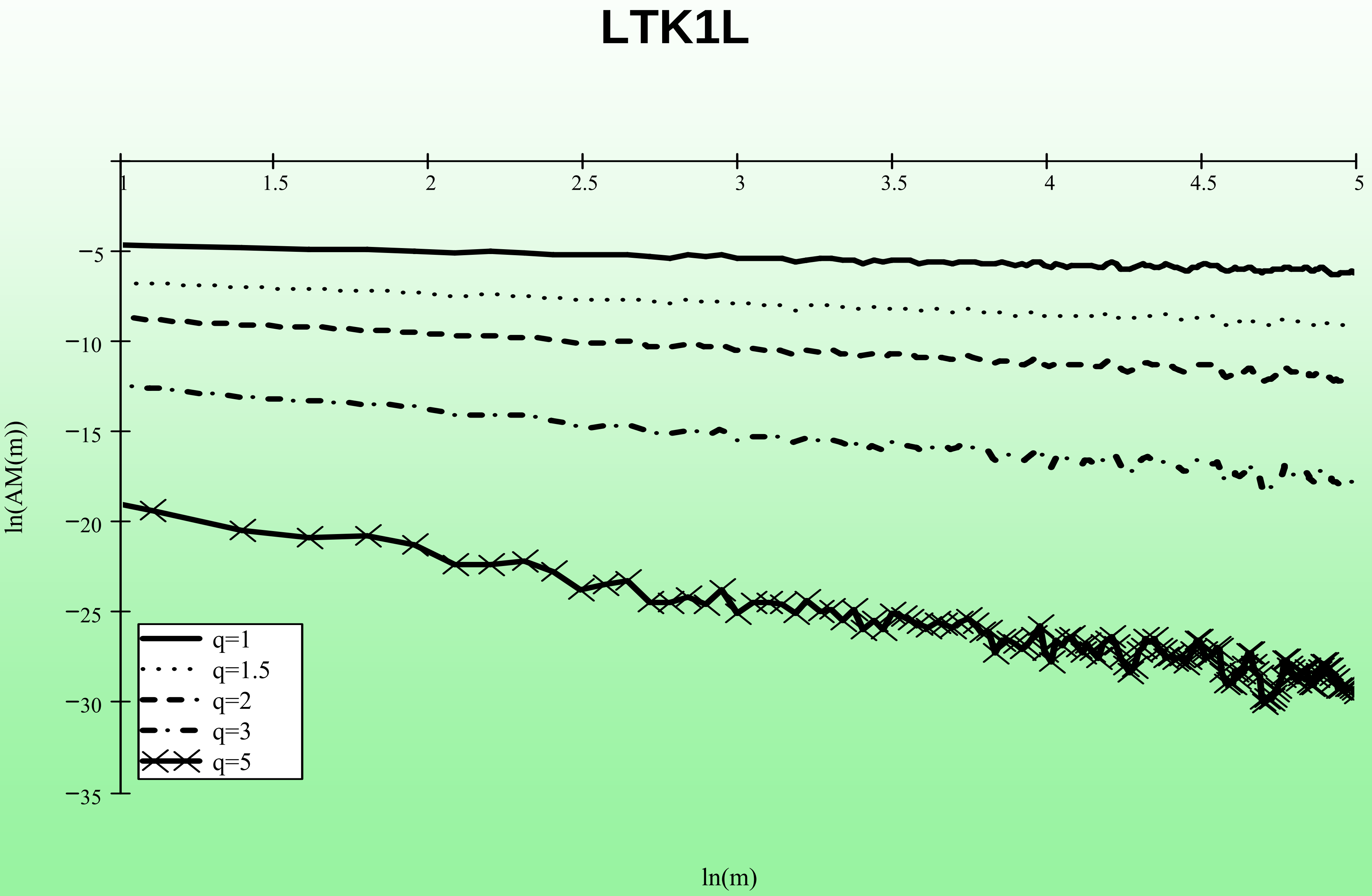
Multifractality and self-similarity



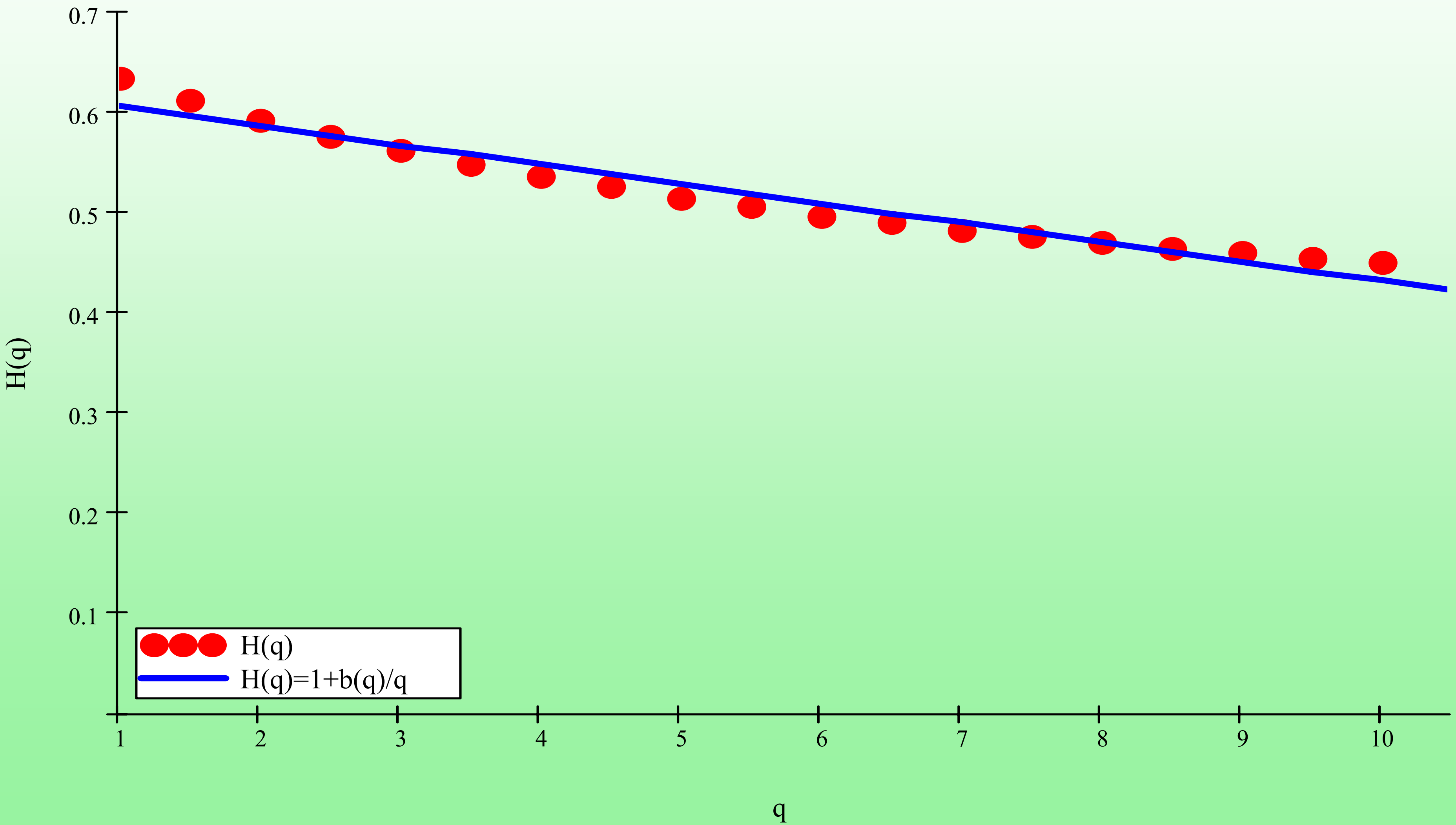
Multifractality and self-similarity



Multifractality and self-similarity



Multifractality and self-similarity

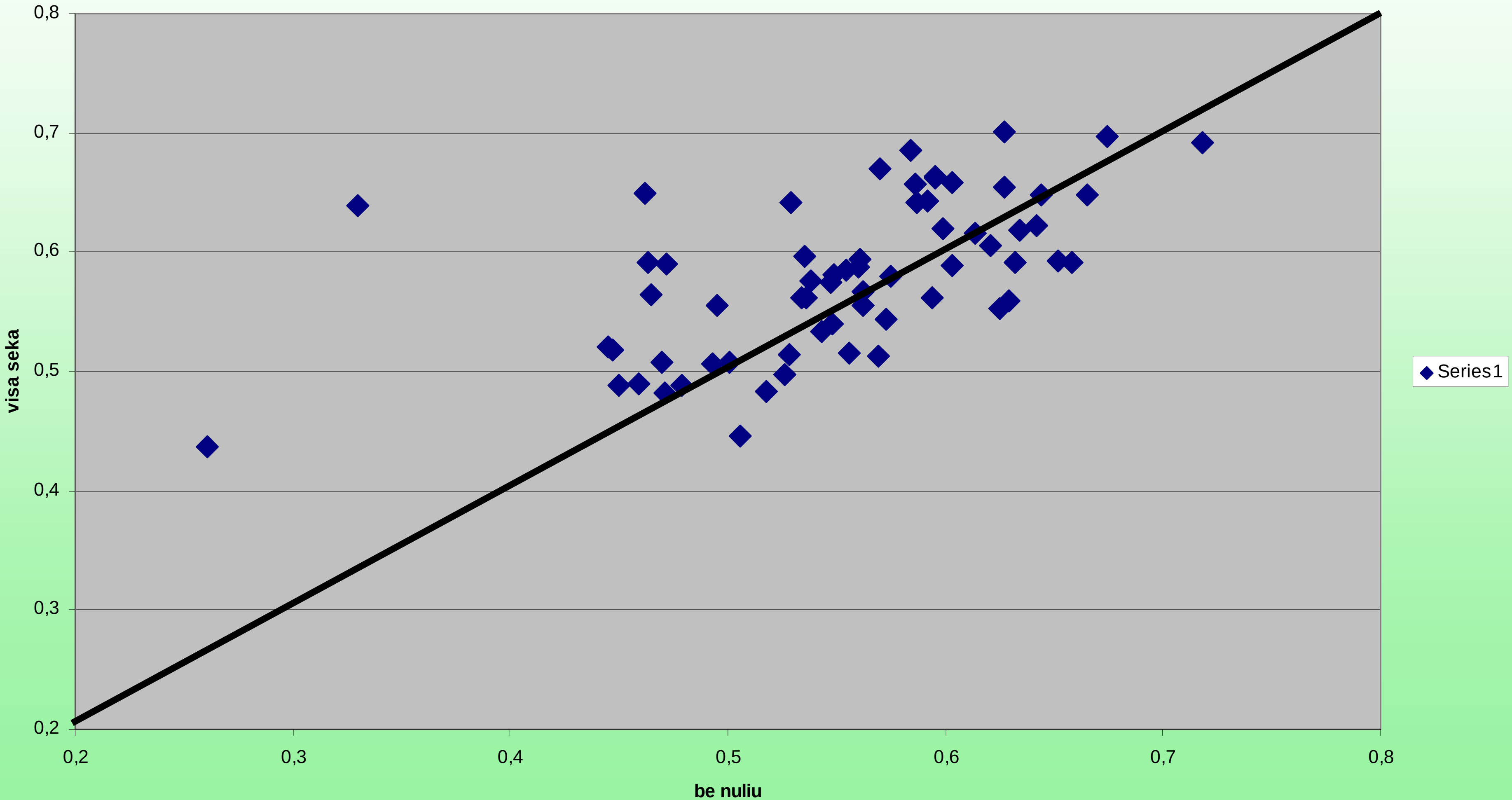


Multifractality and self-similarity
(results)

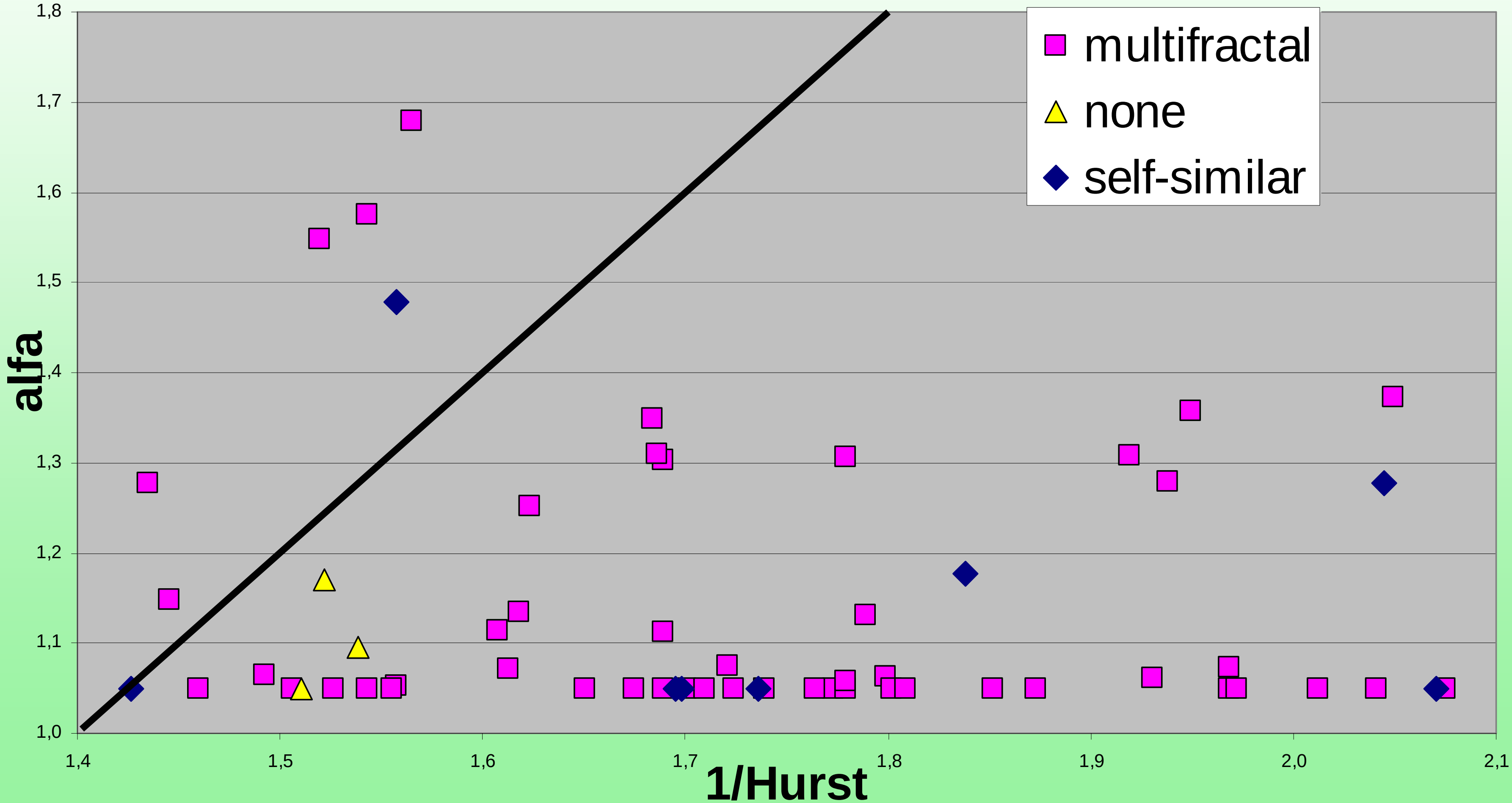
	full series	non–zero series
Self–similar	0+8	12+4
multifractal	42	32
none	15	17

Multifractality and self-similarity

(results, Hurst index)

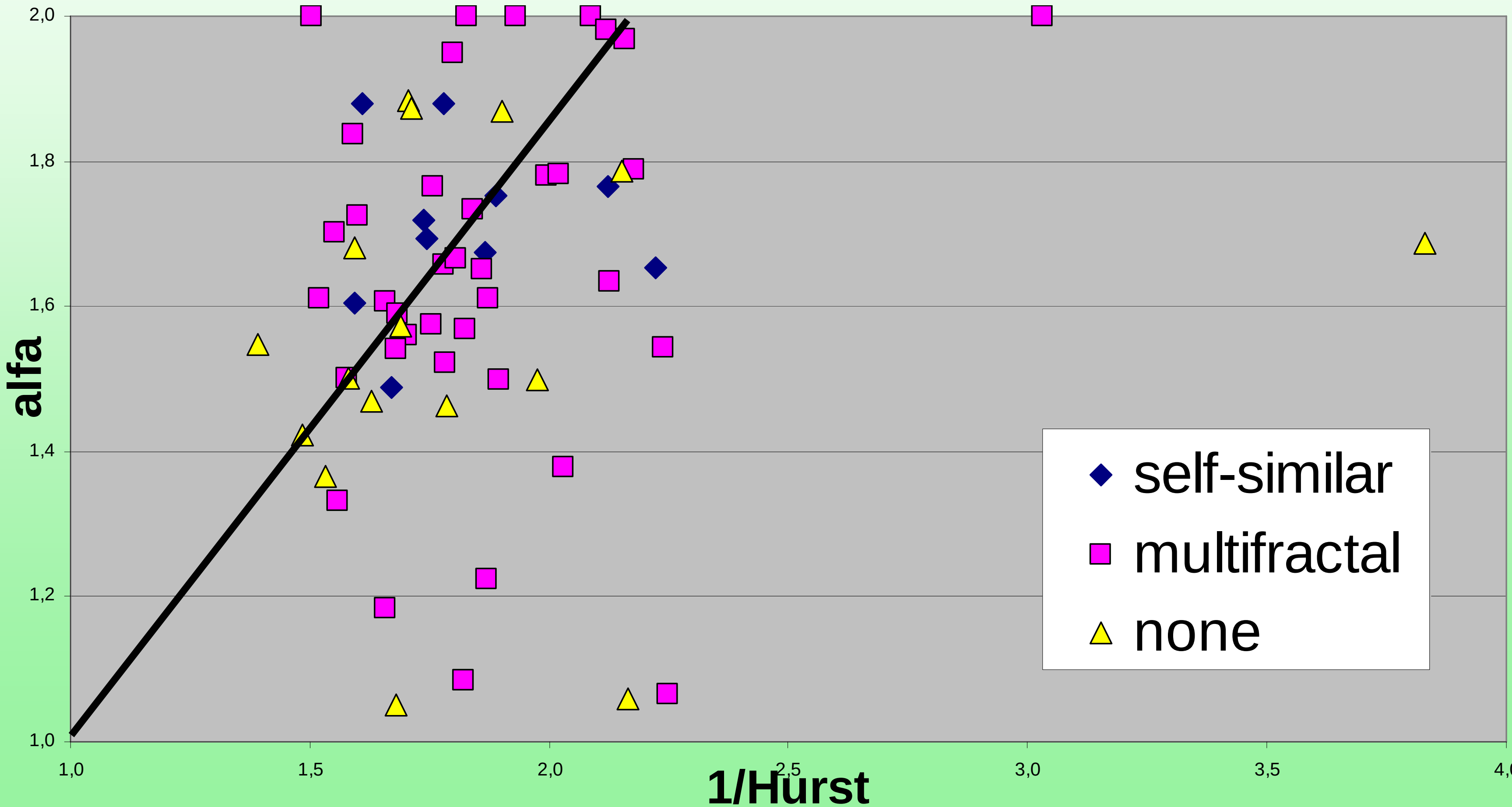


Multifractality and self-similarity
(results, full series)



Multifractality and self-similarity

(results, non-zero series)



Conclusions

- We analyzed all the Baltic Main list and Baltic I-list in period 2000 – 2006.
- Any distribution function not fitted to the empirical data (Anderson–Darling and Kolmogorov–Smirnov goodness–of–fit tests).
- Number of daily zero stock returns for analyzed period differs from 12% to 89% and in average is 52%.
- In “non–zero” series there was found 49 stable series (6 of them were Gaussian) in other 16 cases any distribution function not fitted to the empirical data.

Conclusions

- Analysis of self–similarity in the Baltic States market has not been made yet but it is hampered by short data series.
- Analysis of self–similarity and multifractality has showed that full data series may be analyzed only with more complex model. Full series can not be adequately described by the stable model, but stable model better fits for non–zero series. So mixed model is required to integrate stability and the stagnation phenomenon.

Thanks for listening

☺