Modeling Stock Pinning

Marc Jeannin (King's College), Giulia Iori (City University),
David Samuel (Royal Bank of Scotland)

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What is Stock Pinning?

With Stock Pinning we refer to the tendency of stock's prices to close near the strike price of heavily traded options as the expiration date nears.

A thorough analysis of the pinning effect has been provided by Ni et al. (2005).

The authors show that:

- optionable stocks close near the strike prices on expiration dates, both when the likely delta hedgers have net purchased option positions and net written option positions. There is no corresponding effect for non-optionable stocks.

- As the expiration date approaches, the pinning effect increases when hedgers have net long option positions, but it decreases when delta hedgers have net short option positions.

Thus the authors conclude that

- when traders have net long positions delta hedging does contribute to the pinning.
- On the contrary, when traders have net short positions, the pinning effects is driven by stock manipulation.

Hedging Feedback Effects in Illiquid Markets

- Log-normal model

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

- B&S Δ hedging strategies may induce an additional drift term

$$dS(t) = n\hat{L}S(t)d\Delta(S,t)) + \mu dt + \sigma S(t)dW(t).$$

where $\hat{L}$ is a constant price elasticity (implying linear impact) and n is the open interest on the call option.

- The dynamics of $\Delta(S,t)$ can be derived by using Itô's Lemma

$$d\Delta(S,t) = \frac{\partial\Delta(S,t)}{\partial t}dt + \frac{\partial\Delta(S,t)}{\partial S}dS(t) + \frac{1}{2}\frac{\partial^2\Delta(S,t)}{\partial S^2}dS^2$$

Avellaneda and Lipkin(2003)

- A-L assume that traders believe markets are perfectly liquid. They do not take into account feedbacks effects when rebalancing their portfolio, and assume that the stock price evolves accordingly to the geometric brownian motion. In this case the Delta is the one given by B&S. For a a long call this is

$$\Delta = \partial C/\partial S = -N(d_1).$$

- AL assume (without justifying it) that only the Delta time decay term affects the price dynamics:

$$dS(t) = n\hat{L}\,\frac{\partial \Delta(t, S(t))}{\partial t} S(t)dt + \sigma S(t)dW(t)$$

where they take $\mu = 0$.

AL model generates pinning when traders hedge a long call position. The intuition behind the pinning in this model is clear.

The drift term $\frac{\partial \Delta(t,S(t))}{\partial t}$ is given by

$$\frac{\partial \Delta(t,S(t))}{\partial t} = -\frac{n(h_1)}{\sigma\sqrt{\tau}} \frac{\log y - a\tau}{2\tau},$$

where $\tau = T - t$, $a = r + \sigma^2/2$, $y = S/K$ and

$$h_1 = \frac{\log y + a\tau}{\sigma\sqrt{\tau}}.$$

This term is positive for $y < \exp^{a\tau}$ and negative otherways.

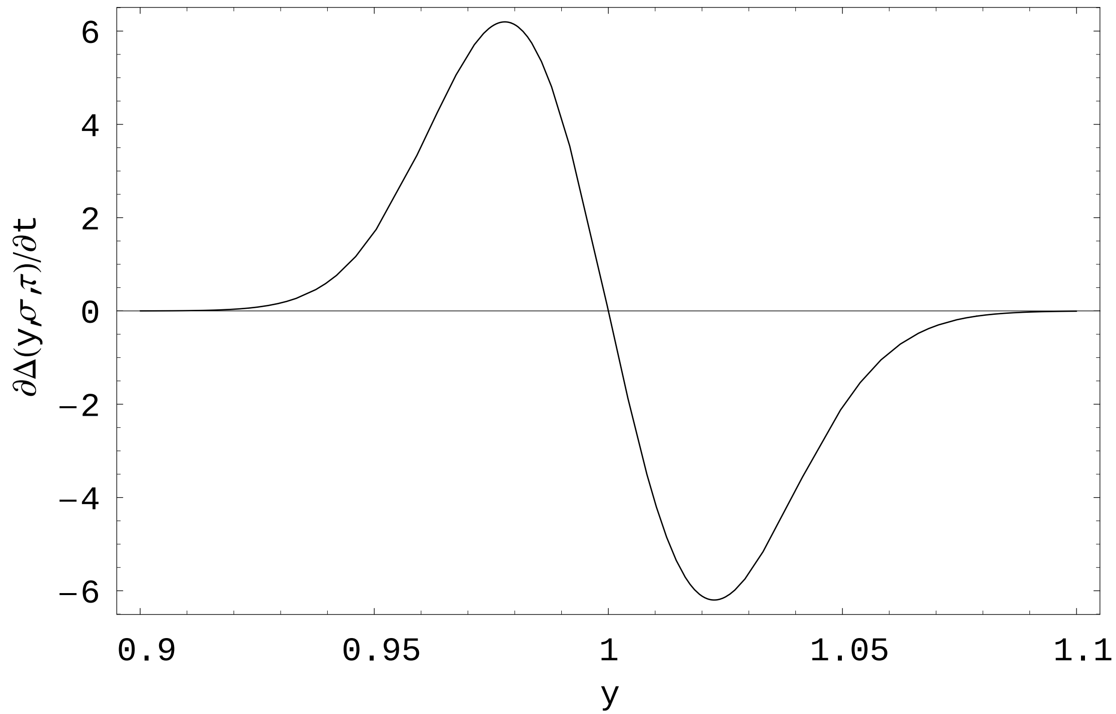


Figure 1: Time decay term as a function of $y = S/K$ for $\tau = 5$ days, $\sigma = 0.16$ and $a = 0$.

Hedging a short position would have the opposite effect pushing the stock price away from the strike.

Frey and Stremme (1997)

The assumption that the time decay term $\frac{\partial \Delta}{\partial t}$ is the leading term in the dynamics of $d\Delta$ is unjustified in A-L.

FS (1997) take all the terms in the delta expansion under consideration and find

$$dS(t) = nLb(t, S(t))S(t)dt + v(t, S(t))S(t)dW(t)$$

with

$$b(t, S(t)) = \frac{1}{1 - n\hat{L}S(t)\frac{\partial \Delta(t,S(t)}{\partial S}} \left(\frac{\partial \Delta(t, S(t))}{\partial t} + \frac{1}{2}\frac{\partial^2 \Delta}{\partial S^2} \frac{\sigma^2 S^2(t)}{(1 - n\hat{L}S(t)\frac{\partial \Delta(t,S(t))}{\partial S})^2} \right),$$

and

$$v(t, S(t)) = \frac{\sigma}{1 - n\hat{L}S(t)\frac{\partial \Delta(t,S(t))}{\partial S}}.$$

F-S introduced their model to explain the volatility smile. Here we show that the model also generates pinning.

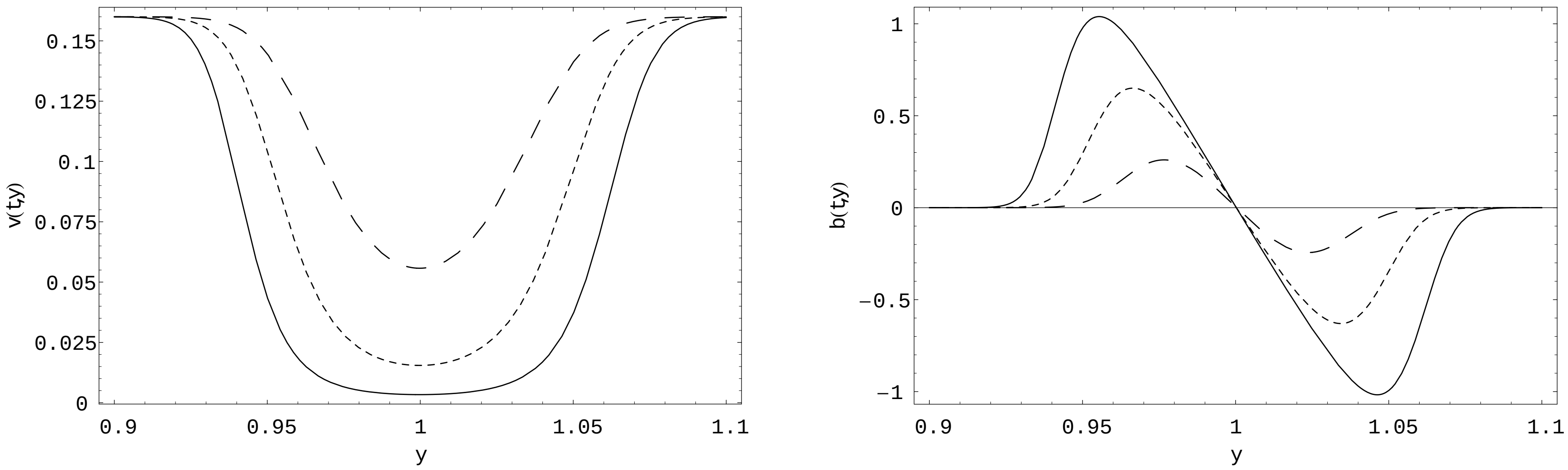


Figure 2: Drift term (left) $b(t,S)$ and volatility term (right) $v(t,S)$ in FS model as a function of $y = S/K$, $\tau = 5$ days and $n\hat{L} = 2.5$(solid), $n\hat{L} = 0.5$(dot), $n\hat{L} = 0.1$(dash).

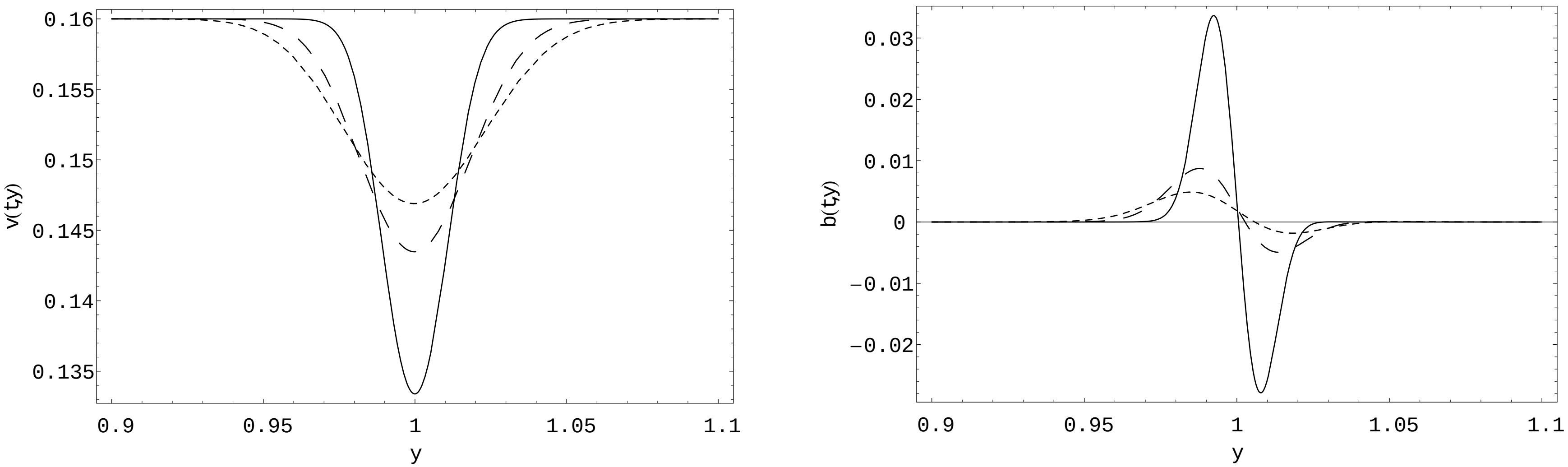


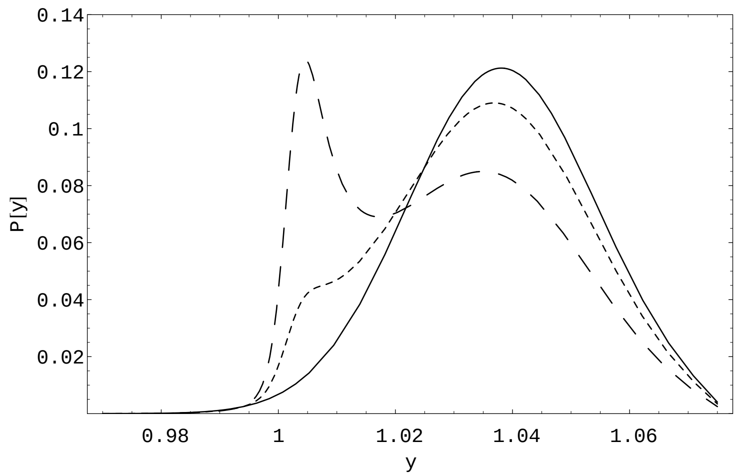
Figure 3: Drift term (left) $b(t,S)$ and volatility term (right) $v(t,S)$ in FS model as a function of y. The price elasticity is $n\hat{L} = 0.05$. The three lines correspond to three different maturity: 1 day before maturity(solid), 3 days before maturity(dash), 4 days before maturity(dot).

We solve the A-L and F-S models numerically. Given eq.(5), the probability density function $p(t, y)$ of being at y at time t satisfies the forward Kolmogorov equation:

$$\frac{\partial p(t,y)}{\partial t} = \frac{1}{2}\frac{\partial^2 p(t,y)}{\partial y^2}v(t,y)^2 - \frac{\partial p(t,y)}{\partial y}b(t,y)$$

with initial condition the delta function $\delta(\tau_0, y_0) = 1$. Equation 6 can be solved using an implicit scheme with an adjusting mesh as we approach maturity due to the singularity at $y = 1$ and $\tau = 0$.

Ni et al. estimate that pinning affects 2% of optionable stocks. Our choices of parameters give comparable values with the empirical result.



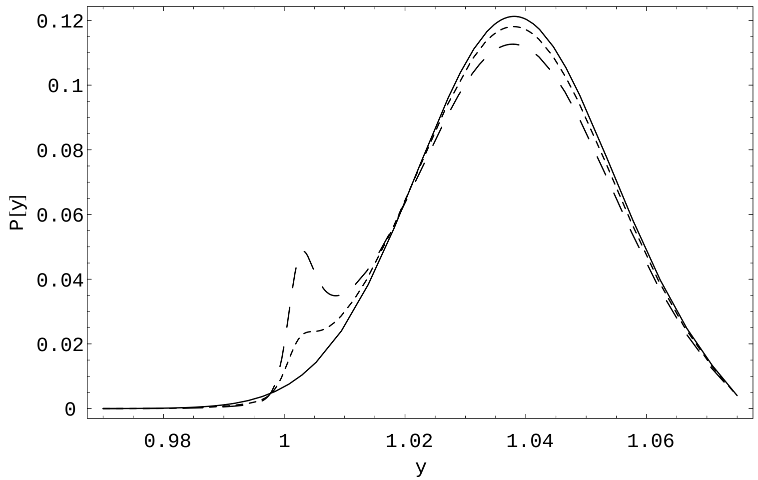


Figure 4: Solution of the Kolmogorov forward equation with initial condition $y_0 = 1.04$, 5 days before maturity for three different hedging positions. (Left) Avellaneda model with $nL = 0$ (solid), $nL = 0.0005$ (small dash), $nL = 0.002$ (large dash). (Right) Frey and Stremme model $nL = 0$ (solid), $nL = 0.002$ (small dash), $nL = 0.003$ (large dash).

Microstructure Model

The main assumptions behind the Frey and Stremme model (as well as of the Avellaneda and Lipkin model) is that the prices are lognormal, rebalancing continuous, the price impact is linear via a constant price elesticity L, and arbitrarily large option positions can always be rehedged (demand and supply always match).

Empirical studies on the NYSE (see for example Lillo et al. (2003)) have shown nonetheless that the price impact function is usually concave, typically well approximated by a function $dp(\omega) \sim \omega^{\alpha}$ where dp is the price change caused by an oreder of volume ω. The exponent α varies from $\alpha \sim 0.5$ to $\alpha \sim 0.2$ depending on stock capitalization.

Furthermore the feedback mechanism in place in these models extrapolates from actual markets condition like the order flow arrival rate and the order book shape.

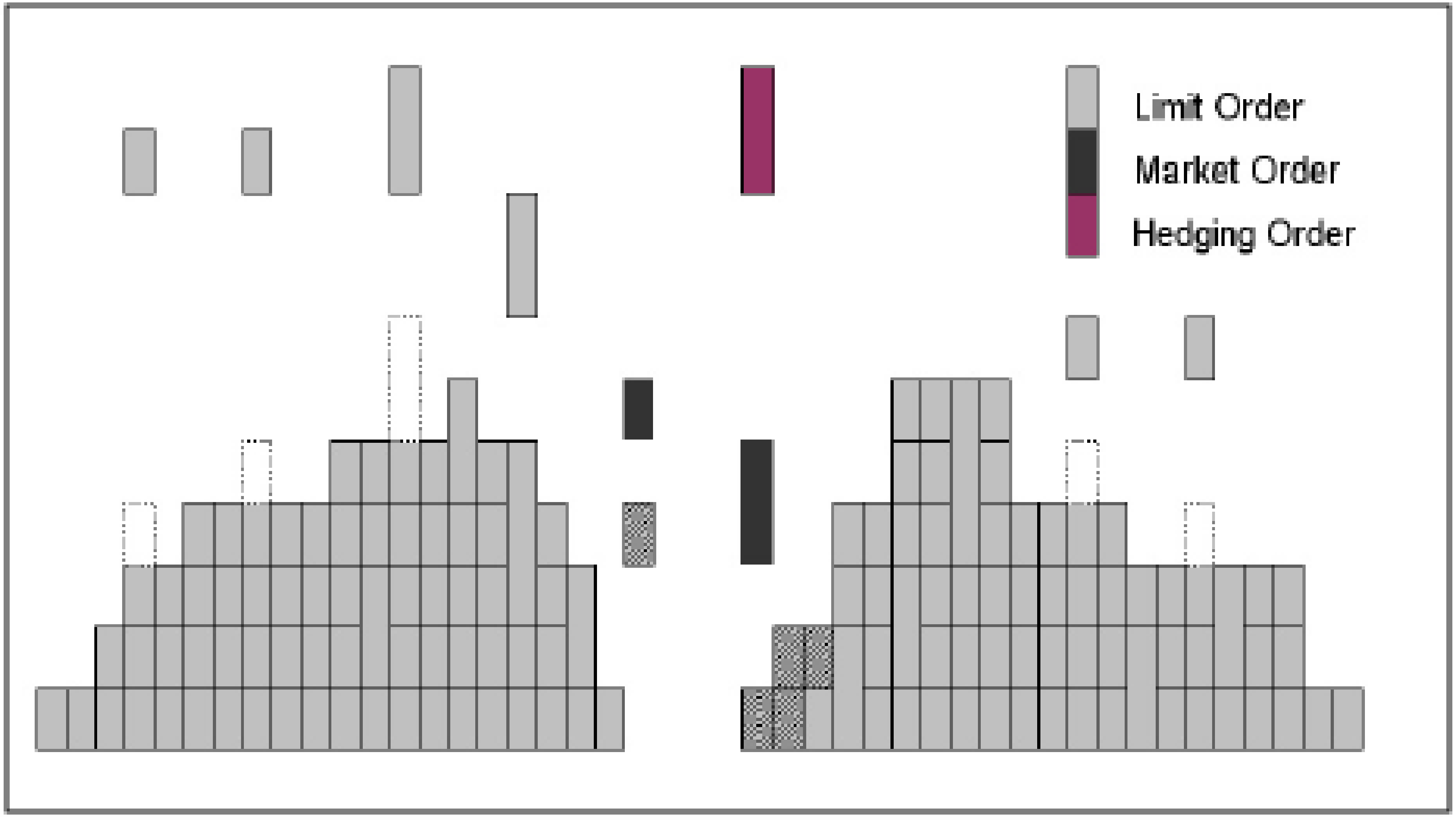
We introduce here a microstructure limit order model, previously introduced in Daniel et al. (2003), where a hedger rebalances his position at discrete times.

The model assumes a simple random order placement of orders. All the order flows are modeled as Poisson processes.

Bids and offers are placed with uniform probability at integer multiples of the tick size Δp on a window sufficiently large around the midpoint.

We assume that

- market orders arrive at a rate of ν shares per unit time, with an equal probability for buy and sell orders.
- limit orders arrive at a rate of α shares per unit price and per unit time.
- limit orders can also be removed spontaneously by being canceled or by expiring: constant rate of δ per unit time.
- The size of the limit and market orders are sampled from a log normal distribution with mean and variance one.


Limit Order
Market Order
Hedging Order

In Daniel et al. (2003) it is shown that two parameters characterize the shape of the book:

- the asymptotic depth α/δ which gives the number of shares per price interval far from the midpoint;
- the parameter $\epsilon = 2\delta/\nu$ which determines the depth at the bid and at the ask. ϵ also determines the price impact function which is linear for $\epsilon > 0.1$ and concave for smaller values of ϵ.

We calibrate the model by assuming that market orders arrive with a frequency of about two a minutes. Hence, we choose $\nu = 0.16$ and $\delta t = 0.08$ minutes. For the price tick we choose $\Delta p = 0.02$. The other parameters were initially set to $\alpha = 0.31$, $\delta = 0.08$, which gives a value of $\epsilon = 1$.

We add one hedger to the model and study if pinning arise.

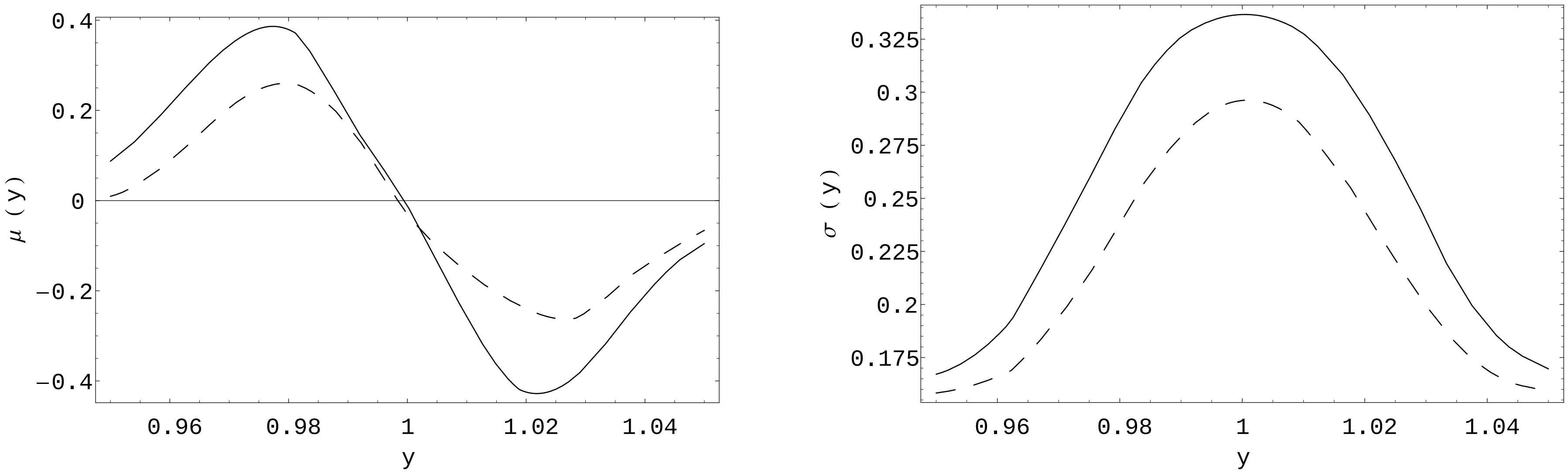


Figure 5: Drift and volatility under microstructure model for 2 differents ϵ: $\epsilon = 0.025$ (dash), $\epsilon = 1.$ (solid) for $n = 300$

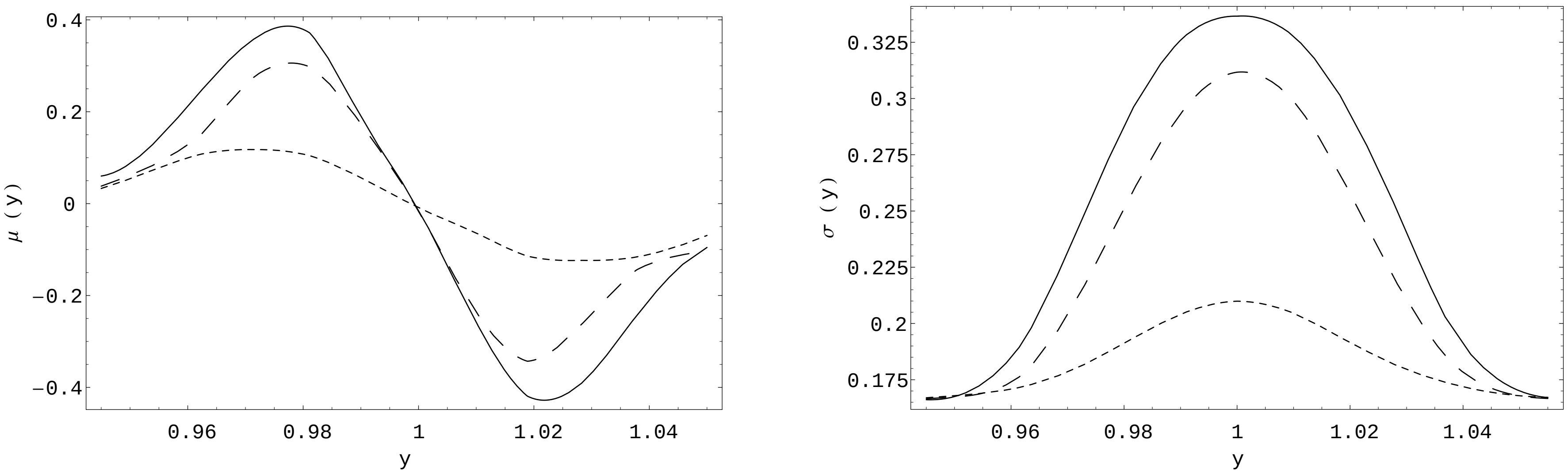
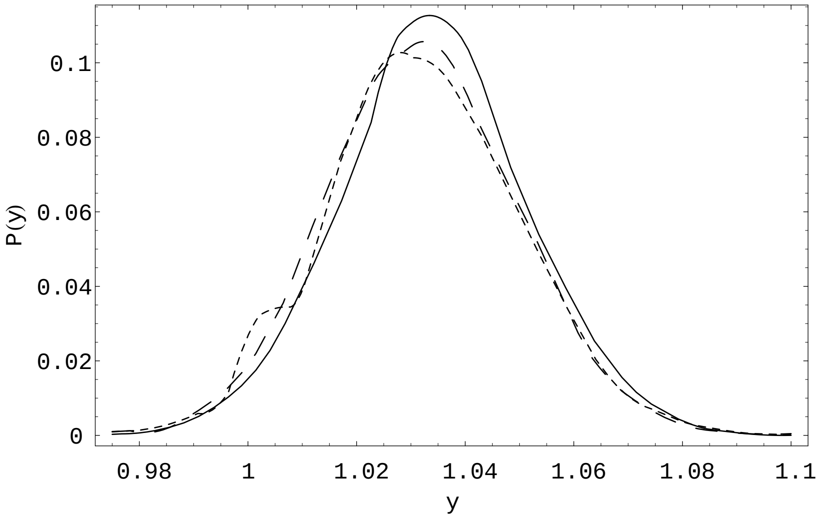


Figure 6: drift and volatility under microstructure model for $\epsilon = 1$ for 3 differents n: $n = 300$ (small dot), $n = 600$ (large dot), $n = 1000$ (solid)



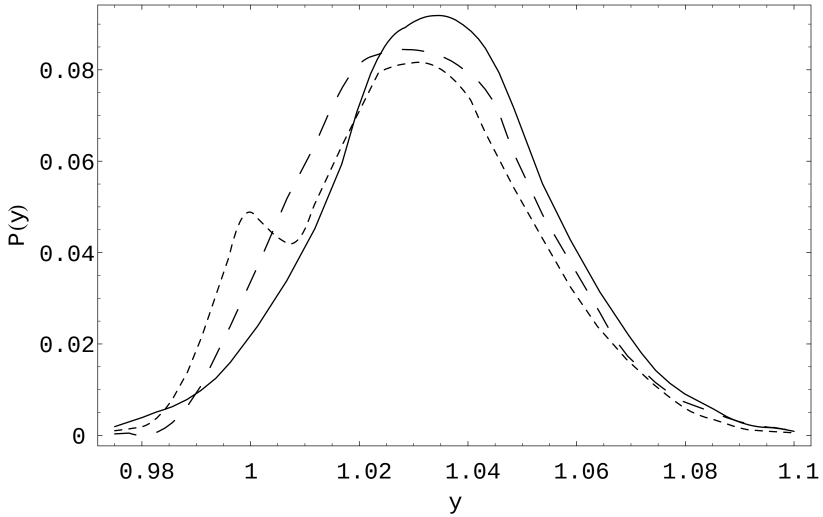


Figure 7: Stock price distribution without the hedger(solid) and with the hedger $n = 500$ (dash) and $n = 1000$ (dot) for 2 different ϵ: $\epsilon = 0.025$ (left), $\epsilon = 1.$ (right)

Conclusion

The main results of this paper are

- We show that the AL model is a simplified form of a previous model introduced by FS
- We study pinning in the FS model, discuss the mechanism that lad to pinning and show that the pinning probabilities are compatible with empirical findings
- We study pinning on a microstructure model which suggest that the volatility increases close to the strike instead of decreasing. This would also imply that the smile would not result from option hedging strategies. More empirical studies are also necessary to clarify the effect of hedging on market volatility, which could also have a role on explaining the volatility smile.

The model also show that pinning is stronger when $\epsilon \sim 1$, i.e. when the price impact is almost linear, thus possibly validating the assumptions of the theoretical models.

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