The internal structure of socium origins from immanent hazy

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The system definition

Definition

The system is a relationship between input and output objects of the system. The input and output objects are assumed to be the given sets $\mathfrak{X}$, $\mathfrak{Y}$. Then the relationship can be specified as a $\mathfrak{S} \subseteq \mathfrak{X} \otimes \mathfrak{Y}$, where $\mathfrak{S}$ means the system. The existence of system $\mathfrak{S}$ is equivalent to selecting some "input-output" pairs.

Multilevel systems I

The additional structure of the system comes from a multilevel decomposition. By this decomposition all elements of the system are distributed across different levels according to their space-time scales and interaction intensities.

Multilevel systems II

Theory of multilevel hierarchical systems (MHST).
This approach follows the methodology of natural hierarchy found by "synergetic" description of biological, social and knowledge systems. MHST was introduced as a general theory of decision making under uncertainty conditions with a given given system task [Mesarovich].

General properties of systems I

- The stratum chosen for the description of a given system depends on tasks and knowledge of observer.
- The functional system descriptions on different levels are not connected to each other generally. Principles and rules, used for the system characterisation on a given level do not follow from the principles and rules used on other levels.
- Each stratum possess its own set of terms, variables, rules and principles. This set creates the language of given stratum. The languages form the semantic relations between any neighboured members of hierarchy.

General properties of systems II

- There exist asymmetrical dependence between conditions of system functioning on different strata. System functioning on given stratum is simultaneously a boundary condition for the lower stratum.

Hierarchical growth I

The system and its interactions are represented by the set of hierarchical coordinates and included into the process of global growth of levels. This process is formulated as a principle of interlayer interaction:

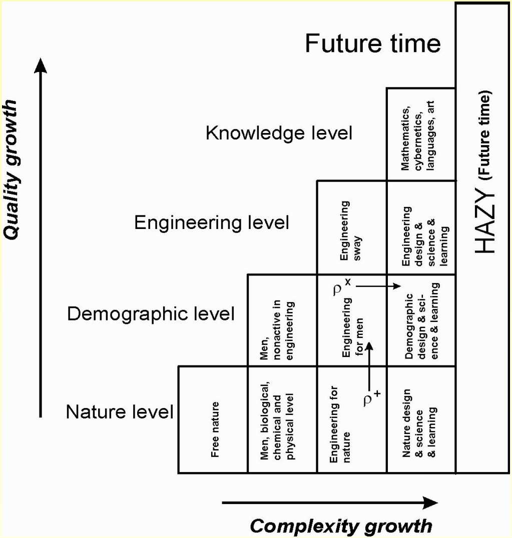
The units of given layer during interaction compose the units of new layer, which, when multiplying, change the constructions of the units of lower levels

Hierarchical growth II

Definition (System development)

The general principle of hierarchical space is formulated as a process of system stratification and self-organization simultaneously. Such understanding is more general than principles of the decision making hierarchical systems or many-echelons systems.

Hierarchical growth III



Structural properties I

- The hierarchical space has the stratified structure. Each stratum has an internal structure as a set of layers, with its own elementary units, interactions, laws and other hierarchical variables.
- Interaction and transition of units along the layer is possible. By this kinds of processes units do not undergo principal transformations of their own structure, they “multiply” or “reproduce” their structure and interactions only. This process is generated by the operator of multiplication ${}^{\times}\rho$.

Structural properties II

- Interaction and transition of units of given stratum through different layers is possible. Under this transition the units undergo a principal change of their own structure and interaction with other units on this and other levels. When the unit moves into the higher levels, it obtains properties of sway, possessing a new "competence". This process is realized by the operator of learning $^{+}\rho$.

Chaotical properties I

In measurement any measurable variable from the full set of variables is the limit of sequence of approximations:

$$A = \lim_{n \to \infty} a_n. \qquad (2)$$

It is true ether for social system, or for the physics processes descriptions. Real measurement can supply one by only limited set of $a_n, n < \tilde{N}$, where $\tilde{N}$ is large enough. This means that A from (2) is “unrealizable” for any $\tilde{N} < \infty$. This situation is very similar to the main idea of non-standard analysis [Robinson]. In this case A is the non-standard number and the halo of monad can be represented as a

$$(A - \lim_{n \to \tilde{N}} a_n) =^G A \qquad (3)$$

Chaotical properties II

For the dynamical hierarchical system elements of the system also can be represented as a mathematical objects of non-standard nature in sense of Robinson. The set of this objects (hyperreal numbers axis) $\mathbb{G}$ is the expansion of real numbers field $\mathbb{R} \in \mathbb{G}$.

Chaotical properties III

Definition

Let $\mathfrak{L}$ is the language with own semantic, true in the universe U and $\models \alpha$ means that sentence α is truly in the universe U. Let α is the sentence in $\mathfrak{L}$. Then

$$* \models^* \alpha \text{ then and only then, if } \models \alpha$$

There we can buld the stahdard universe U and non-standard universe ${}^*U$. This extended set $\mathbb{G} \in^* U$ has non-archimedian character and according the Los theorem we can formulated *the transfer principle or standartization principle*.

Free will I

For the social system the free will of elementary units makes any reference to infinite number of measurements or their infinite accuracy very problematic.
The system can choose different way according own will:

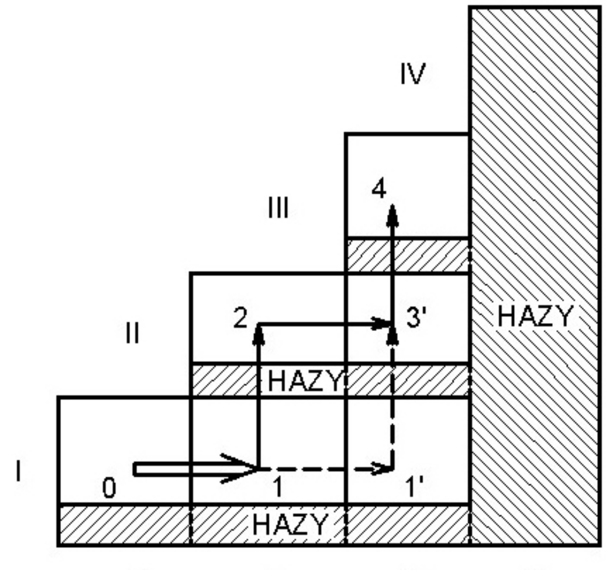
$$\rho^{+}\rho^{+}\rho^{+},$$

or operators

$$\rho^{+}\rho^{\times}\rho^{+}.$$

for the process $(1 \Rightarrow 2 \Rightarrow 3' \Rightarrow 4)$.

Free will II



Social groups dynamics I

Let the system state be represented by two-component value and the initial $a_{[1]}$ and final $a_{[2]}$ states, where

$$a_{[1]} = \begin{pmatrix} {}^{st}a_{[1]} \\ {}^{h}a_{[1]} \end{pmatrix}, \quad a_{[2]} = \begin{pmatrix} {}^{st}a_{[2]} \\ {}^{h}a_{[2]} \end{pmatrix}, \tag{6}$$

We believe that standard and nonstandard parts of given state level are supposed to be independent.

$$\rho^{+} = \begin{pmatrix} \rho^{+}_{[st\leftarrow st]} & \rho^{+}_{[st\leftarrow h]} \\ \rho^{+}_{[h\leftarrow st]} & \rho^{+}_{[h\leftarrow h]} \end{pmatrix}, \tag{7}$$

$$\rho^{\times} = \begin{pmatrix} \rho^{\times}_{[st\leftarrow st]} & \rho^{\times}_{[st\leftarrow h]} \\ \rho^{\times}_{[h\leftarrow st]} & \rho^{\times}_{[h\leftarrow h]} \end{pmatrix}. \tag{8}$$

Social interpretation I

Consider now arising of social groups along the path $[1 \to 1' \to 2]$. In this case groups are self-organized from the ensemble of typical individuals

$$\begin{pmatrix} {}^{st}a_{[2]} \\ {}^{h}a_{[2]} \end{pmatrix} = \begin{pmatrix} \rho^{+}_{[st\leftarrow st]} & \rho^{+}_{[st\leftarrow h]} \\ \rho^{+}_{[h\leftarrow st]} & \rho^{+}_{[h\leftarrow h]} \end{pmatrix} \cdot \begin{pmatrix} \rho^{\times}_{[st\leftarrow st]} & \rho^{\times}_{[st\leftarrow h]} \\ \rho^{\times}_{[h\leftarrow st]} & \rho^{\times}_{[h\leftarrow h]} \end{pmatrix} \begin{pmatrix} {}^{st}a_{[1]} \\ {}^{h}a_{[1]} \end{pmatrix}. \quad (9)$$

Social interpretation II

Standard part of the group ${}^{st}a_{[2]}$:

$$
{}^{st}a_{[2]} = \left(\rho^{+}_{[st\leftarrow st]}\rho^{\times}_{[st\leftarrow st]} + \rho^{+}_{[st\leftarrow h]}\rho^{\times}_{[h\leftarrow st]}\right) {}^{st}a_{[1]} + \left(\rho^{+}_{[st\leftarrow st]}\rho^{\times}_{[st\leftarrow h]} + \rho^{+}_{[st\leftarrow h]}\rho^{\times}_{[h\leftarrow h]}\right) {}^{h}a_{[1]}. \quad (10)
$$

First summand represents transition from amorphous collective of individuals to social group (primitive human herd transforms to tribe). Second term describe the process of social learning and creation of the culture of given group. Third summand corresponds to the multiplication of certain individual feature across the socium resulting in social type with formal organization (Ku-Klux-Klan). The last term designates the objectivation of collective unconscious of the group (myth, ritual).

Social interpretation III

Non-standard part of ${}^{h}a_{[2]}$ has the form

$$ {}^{h}a_{[2]} = \left(\rho^{+}_{[h\leftarrow st]}\rho^{\times}_{[st\leftarrow st]} + \rho^{+}_{[h\leftarrow h]}\rho^{\times}_{[h\leftarrow st]}\right) {}^{st}a_{[1]} + \left(\rho^{+}_{[h\leftarrow st]}\rho^{\times}_{[st\leftarrow h]} + \rho^{+}_{[h\leftarrow h]}\rho^{\times}_{[h\leftarrow h]}\right) {}^{h}a_{[1]}. \quad (11) $$

First term corresponds to unstable association with legalized statute (public organization without members, internet forums, humanitarian and welfare funds). Second term corresponds to unstable groups of revisionists and innovators. Third term corresponds to social groups which really exist but do not form self-consciousness (for example, age-specific generations). The last summand represents such unstable and hardly observable entities as people soul, collective self-suggestions etc.

Social interpretation IV

In both paths only first terms in standard parts corresponds to commonly used models of sociodynamics. Additional terms in the proposed approach make it possible to account for mental factors in such models conserving their mathematical structure and increasing the state space dimension of them only.

Iterated function system I

Let us investigate an iterated function system (IFS) [Barnsley]. An IFS consist of a certain number of k function $F_i,\ i = 1, 2, \ldots, k$, which act randomly with given probabilities $p_i,\ i = 1, 2, \ldots, k$. IFS can be considered as a combination of deterministic and stochastic dynamics. This IFS reduce system evolution to the discrete consequences of stationary states. It can be shown that IFS generates unique invariant measures, in general case fractal.

Iterated function system II

The IFS $F = \{F_i, p_i, i = 1, 2, \ldots, k\}$ generates a Markov operator V, acting on space *M(X)*:

$$(V\nu)(B) = \sum_{i=1}^{k} \int\limits_{F_i^{-1}(B)} p_i(\lambda)\, d\nu(\lambda). \qquad (12)$$

The space *M(X)* is the space of all probabilities X, $\nu \in M(x)$ and B is the measurable subset X. Markov operators of this form describe the evolution of probability measures under the action $F = \{F_i, p_i, i = 1, 2, \ldots, k\}$, and the IFS is the continuous random function.

Iterated function system III

The Markov process can be built in the following way. Take the probability measure on the Ω:

$$P_x(i_1, i_2, \ldots, i_n) := \\ = P_x\Big(\{\omega \in \Omega : \omega(j) = i_j, j = 1, \ldots, n\}\Big) = \\ := p_{i_1}(x)\, p_{i_2}(F_{i_1}(x)) \ldots \times p_{i_n}\Big(F_{i_{n-1}}(\ldots(F_{i_1}(x)))\Big), \quad (13)$$

where $x \in X,\ i_j = 1, \ldots, k,\ j = 1, \ldots, n;\ n \in N$.
Ω produced the code space,$\Omega = \{1, \ldots, k\}^{\mathbb{N}}$.
For an IFS which fulfils additional assumption, and expression (12—**??**), there exists a unique invariant probability μ satisfying the equation:

$$V\mu = \mu.$$

Entropy and optimal evolution I

For the code space $\Omega = \{1, \ldots, k\}^{\mathbb{N}}$ we can determine measure of probability P_μ as:

$$P_\mu(i_1, \ldots, i_n) := P_\mu(\{\omega \in \Omega : \omega(j) = i_j, \quad j = 1, \ldots, n\}) = \int_X P_x(i_1, \ldots, i_n)\, d\mu(x) \qquad (14)$$

for $i_j = 1, \ldots, k,\ j = 1, \ldots, n;\ n \in (N)$.
We obtain for the partial entropy a discrete set of variables:

$$H(n) = -\sum_{i_1, \ldots, i_n = 1}^{k} P_\mu(i_1, \ldots, i_n) \ln P_\mu(i_1, \ldots, i_n). \qquad (15)$$

Entropy and optimal evolution II

Relative entropy is given by the expression:

$$G(1) := H(1), \quad G(n) = H(n) - H(n-1) \text{ for } n > 1. \quad (16)$$

The KS entropy is the exact value of the non-standard number because the range of integration in expression (14—16) is the non-standard number.

$$K_1 = \lim_{n\to\infty} G(n) = \lim_{n\to\infty} \frac{F(n)}{n}.$$

Entropy and optimal evolution III

If we introduce the Rényi entropy by standard manner analogously to the Rényi dimension we receive:

$$K_\beta = \frac{1}{1-\beta} \ln \sum_i (p_i)^\beta = \lim \sup_{n\to\infty} \frac{1}{1-\beta} \times$$
$$\times \ln \left[\sum_{i_1,\ldots,i_n}^{k} P_\mu(i_1,\ldots,i_n) \left(\frac{P_\mu(i_1,\ldots,i_n)}{P_\mu(i_1,\ldots,i_{n-1})} \right)^{\beta-1} \right]. \quad (17)$$

In (17) β is the free parameter. The Rényi entropy is equal to the topological entropy if $\beta = 0$.

Entropy and optimal evolution IV

For the distribution (14) an escort distribution can be introduced. Let us present this distribution (14) in the form:

$$P_\mu\left(i_1,\ldots,i_n\right)=\exp\left(-b_\mu\left(i_1,\ldots,i_n\right)\right)\times \times\sum_{i_1,\ldots,i_n}P_\mu\left(i_1,\ldots,i_n\right)= =\sum_{i_1,\ldots,i_n}\exp\left(-b_\mu\left(i_1,\ldots,i_n\right)\right)=1, \quad (18)$$

then the escort distribution takes the form:

$$p_\mu=\frac{\left(P_\mu\right)^\beta}{\sum\limits_{i_1,\ldots,i_n}\left(P_\mu\right)^\beta},\quad \sum_{i_1,\ldots,i_n}\left(p_\mu\right)=1.$$

Energy and temperature I

If we use the representation (18) the escort distribution for the system is a Gibbs canonical distribution:

$$p_\mu = \exp\left[\beta\left(F\left(\beta\right) - b_\mu\right)\right],$$

$$F\left(\beta\right) = -\frac{1}{\beta}\ln\sum_{i_1,\ldots,i_n}\left(-\beta b_\mu\right), \qquad (19)$$

where b_i plays a role of effective Hamilton function, $F\left(\beta\right)$ is the corresponding free energy, $1/\beta$ is the effective temperature.

Energy and temperature II

From (19), taking into account expression (17), follows a link between the free energy and the Rényi entropy:

$$F(\beta) = -\frac{1}{\beta} \ln \sum_i \exp(-\beta b_i) =$$
$$= -\frac{1}{\beta} \ln \sum_i \exp(P_i)^\beta = -\frac{1-\beta}{\beta} K_\beta [P]. \quad (20)$$

$$F(\beta = 1) = -\ln \sum_i P_i = 0,$$

and additionally $p_i = P_i$.

Energy and temperature III

The Rényi entropy can be taken as a possible measure of the chaotical state of the system. From the physical meaning of chaos and according to the main principles of non-standard analysis distribution (18) forms the monad of standard number, and the standard number corresponds to the KS entropy. The standard part of entropy is the function of fixed point A and KS entropy is a characteristic of the equilibrium state because the measure μ is attractive. The halo of monad of non-standard numbers in one-dimension code space :

$$\Gamma = \lim_{\beta \to 1} \left(K_\beta - K_1 \right).$$

The value Γ represents the internal indeterminacy of the appropriate dynamical system.

Conclusion I

- Chaos growth by the hierarchical evolution
- Self-organisation follow from hazy structure
- Physical definition of optimal evolution
- Entropy as a control parameter of evolution
- Energy and temperature of hierarchical dynamical systems

Conclusion II

Vilnus, 25 years ago...

THANK FOR YOUR ATTENTION