Dynamical complex networks of rational economical agents

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Vilnius May 2006

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Santa Fe Institute preprint 2005-06-022
cond-mat/0504190

Motivation

Old and fundamental question in economics:

Why is there cooperation in a society if for the individual it is better not to cooperate?

This is a complex systems question: How does there arise the 'quality' of cooperation from interactions on non-cooperative elements?

Formal framework to treat questions of cooperation is game theory

John von Neumann and Oskar Morgenstern: Theory of Games and Economic Behavior (1944); John F. Nash: Nobel price (1994)

The prisoners dilemma

- Game between 2 persons with 2 possible actions: cooperate or defect
- Payoff-Matrix for player i:

		Player j C	Player j D
Player i	C	R	S
	D	T	P

$$P_{ij} = \begin{pmatrix} 3 & -1 \\ 4 & 0 \end{pmatrix}$$

R ... Reward, S ... Sucker's payoff, T ... Temptation, P ... Punishment

- Two inequalities must hold: $S < P < R < T$ and $T + S < 2R$
- Dilemma: Individual rationality leads to defection

So why is there cooperation ?

Classical answer: There is an external entity which effectively changes the payoff matrix. E.g. State punishes non-cooperative behavior, legal protection etc. (Buchanan 1975)

Unsatisfactory since usually do not cooperate because we are forced but because want to

Clever way out: Iterated PD: people 'learn' that cooperation in the long run leads to higher payoffs. Play game many times – introduce memory base decisions whether to cooperate or not on experience (Weibull, Axelrod, Sigmund,...)

Usually implemented on fixed topologies regular lattices or any type of networks, most of the time by strategies of imitation (Nowak 1992). Less work done with variable linkage

- Missing: IPD on dynamic networks under *full rationality* and *local information*

Missing aspect: freedom of choise

Markets at a basic level: Individuals can choose with whom to interact

Question: How do markets and societies act when interaction topology is dynamic?

- Will they cooperate?
- Will there be self organization of interaction patterns?
- What do interaction networks look like?

Iterated PD on networks

- Iterated PD played by N players on a dynamic network
- Each agent i plays with all of his neighbors $\tilde{N}(i,t)$ at time t
- His action $a_i(t)$ chosen at time t is the same for all co-players
- $a_i(t)$ is either $a_C = (1,0)$ for cooperation or $a_D = (0,1)$ for defection
- Payoff of player i at timestep t:

$$P_i(t) = \sum_{j \epsilon \tilde{N}(i,t)} a_i(t) P_{ij} a_j(t) \tag{1}$$

(Note formal analogy to spin systems if $P_i(t)$ is associated with energy)

Assumptions

- Players update strategy (with whom, C or D) with probability p_u
- Information horizon of player is 1 (locality, knowledge about neighbors)
- Links may only be established to next to nearest neighbors (recommendation)
- At maximum α / β links are broken / built up per player per time (limit of resources)
- Players act fully rationally and have expectations one timestep into future (martingale)

Model I

- Player i calculates his expected payoff for cooperation and defection, where $\Delta\bar{P}_i(t+1)$ is the payoff expected from new links:

$$\bar{P}_i^c(t+1) = \sum_{j \epsilon \tilde{N}_c(i,t+1)} a_c P_{ij} \bar{a}_j(t+1) + \Delta\bar{P}_i(t+1)$$
$$\bar{P}_i^d(t+1) = \sum_{j \epsilon \tilde{N}_c(i,t+1)} a_d P_{ij} \bar{a}_j(t+1) \quad (2)$$

- Assume that i's expectation for the strategy of player j in the next timestep is $\bar{a}_j(t+1) = a_j(t)$

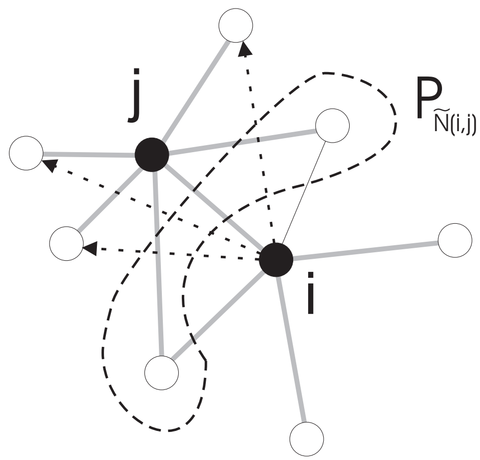
- $\tilde{N}_c(i, t+1)$ is the neighborhood at $t+1$, where α links to defecting agents have been canceled

Model II

- $\Delta \bar{P}_i(t+1)$ can be estimated by considering common neighbors $\tilde{N}(i,j)$ and the number of neighbors not in common N_{nc} at time t:

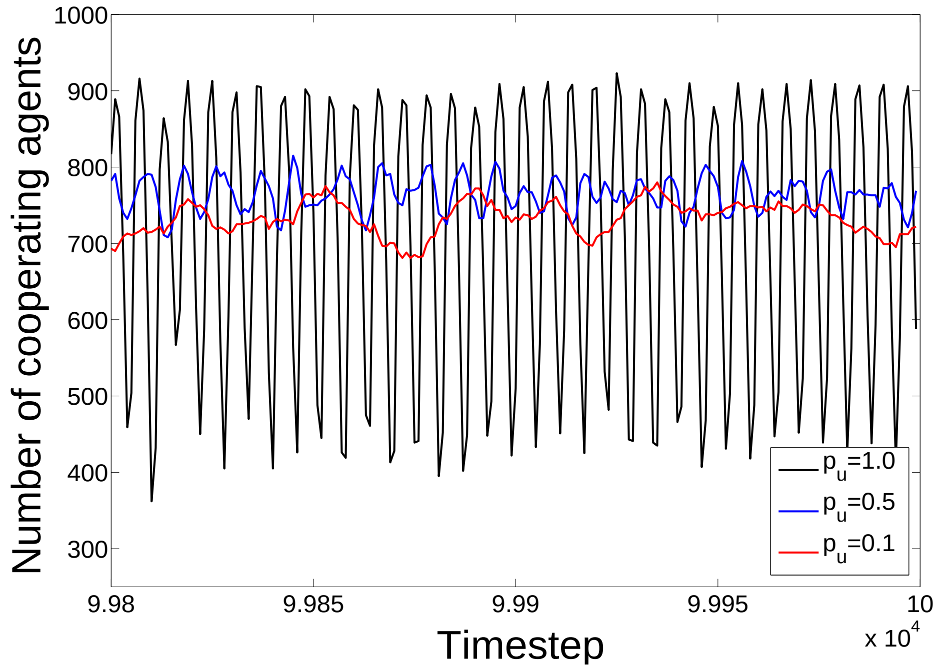
$$\Delta \bar{P}_i(t+1) = (P_j(t) - P_{\tilde{N}(i,j)}(t))\frac{\beta}{N_{nc}} \quad (3)$$

- β/α is maximal number of links agent i can build up / break per timestep



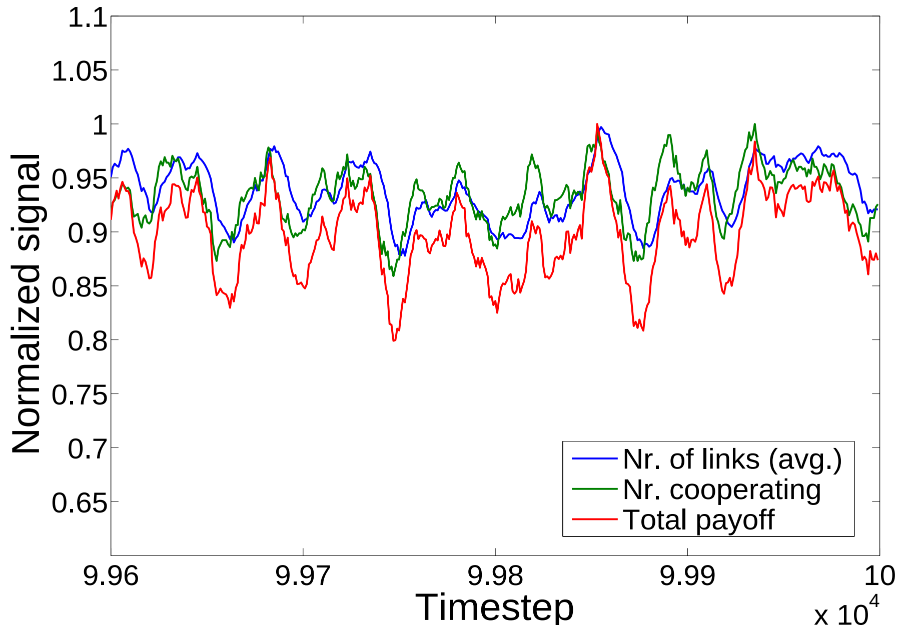
Results – cooperation I

- For given α and β behavior depends on update probability p_u

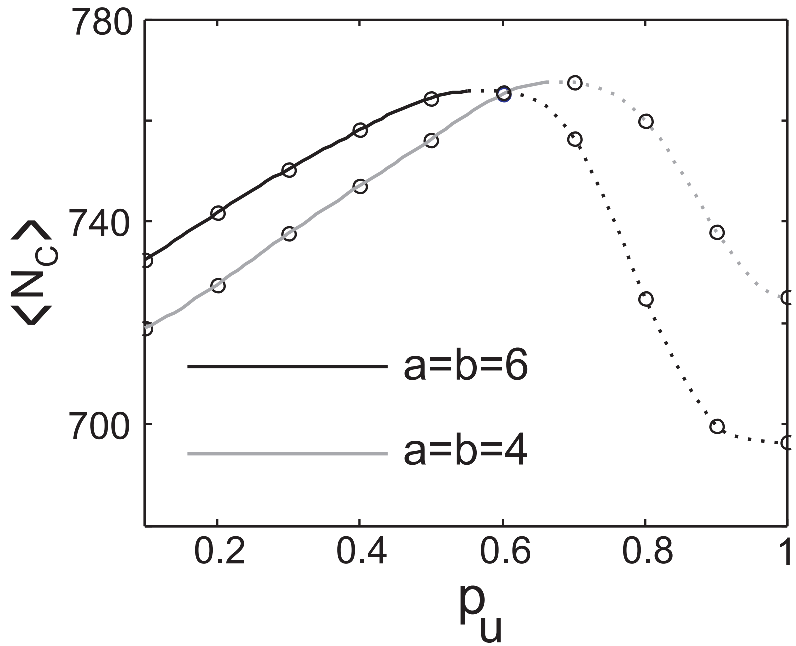


Results – cooperation II

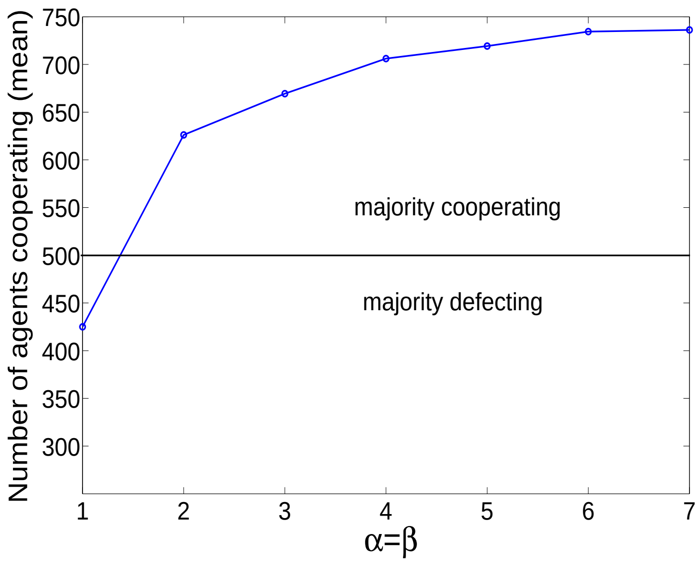
- Number of cooperating agents, average number of links and total payoff $P_{tot}(t) = \sum_i P_i(t)$ are 'co-integrated'



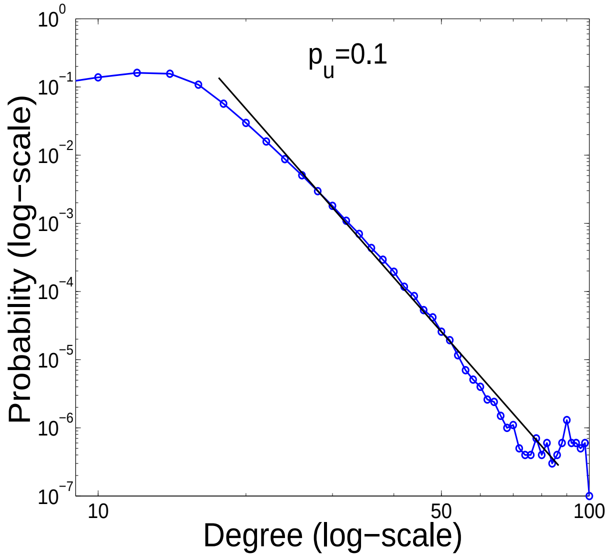
Dependence on agility

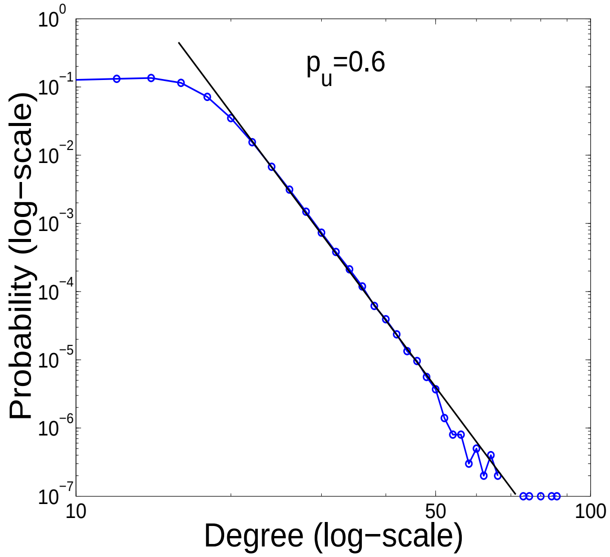


Dependence on resource parameters α and β

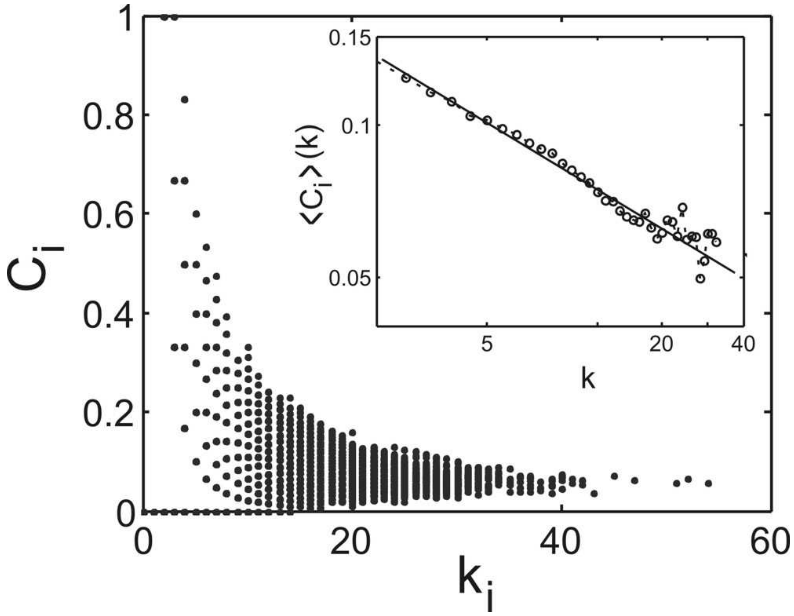


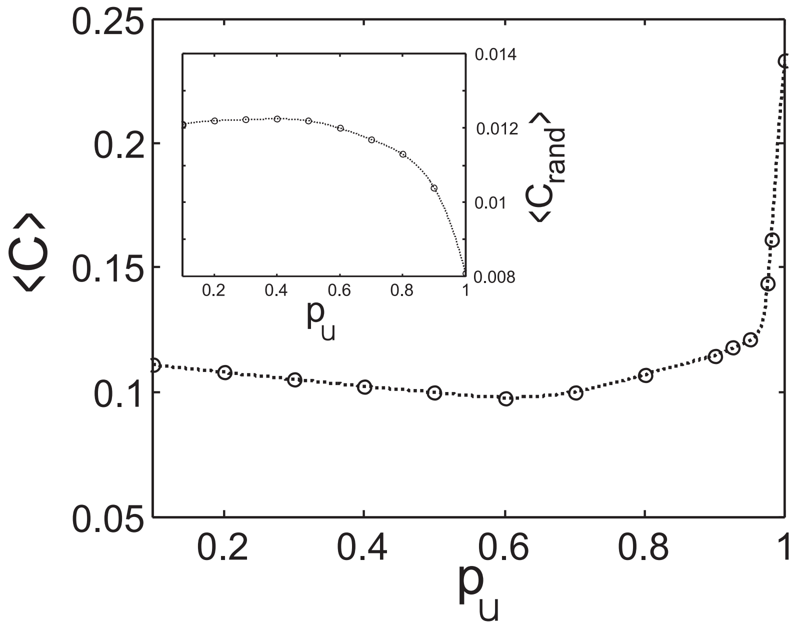
Resulting networks – Degree distribution





Resulting networks – Clustering coefficient





Conclusion

• Cooperation emerges in the prisoners dilemma if freedom of choice of partners is allowed, even with *local information* and full rationality of the agents

• No imitation, no external sanctions and no global information needed

• There is an cooperative optimum in agility (update probability)

• The more resources (α and β) the higher the global optimum

• The emerging cooperation networks are scale free over wide parameter regions, and show non-trivial clustering