

Physics of Risk

13-16 May 2006
Vilnius

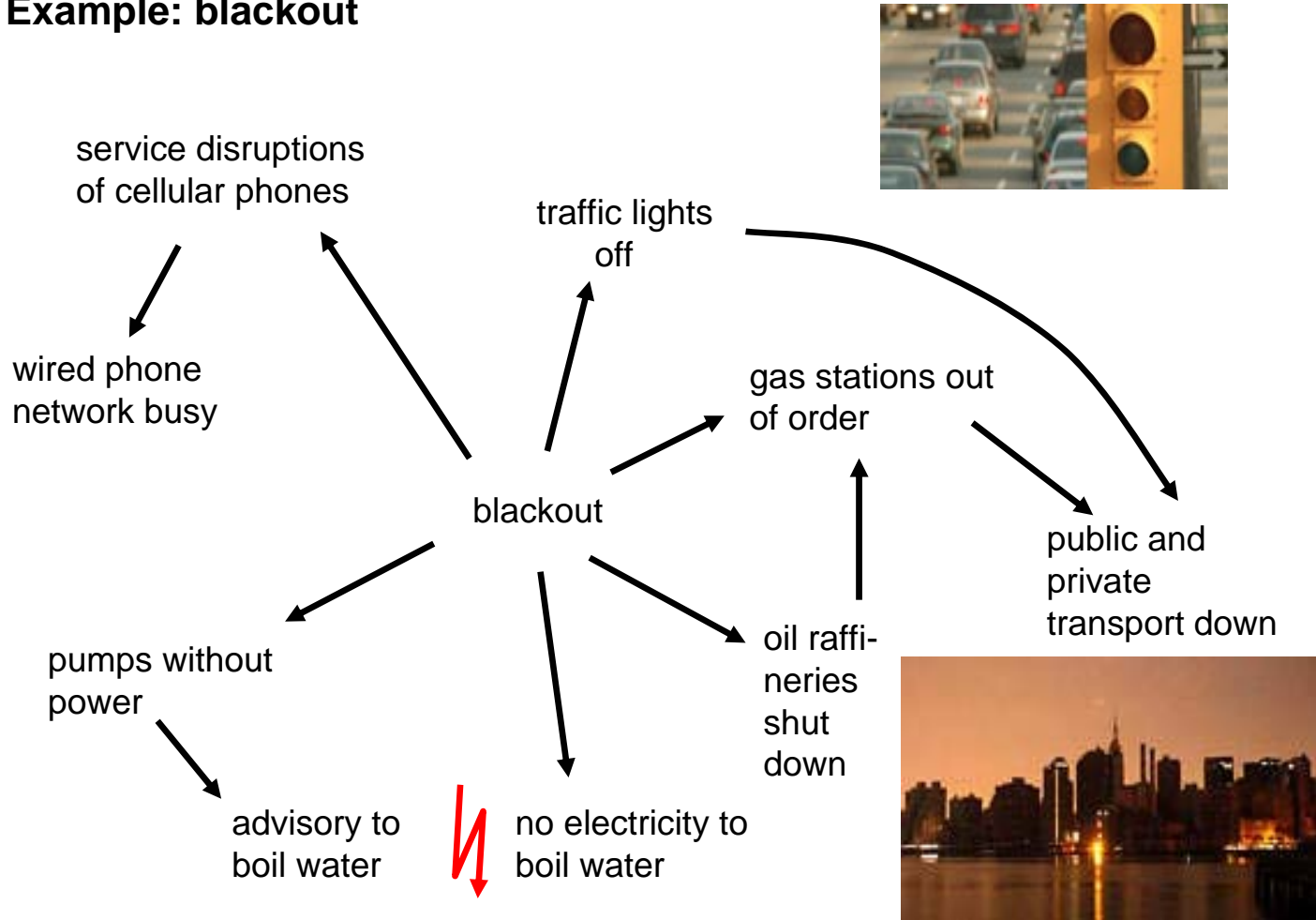
Disaster spreading in complex networks

Karsten Peters, Lubos Buzna, Dirk Helbing

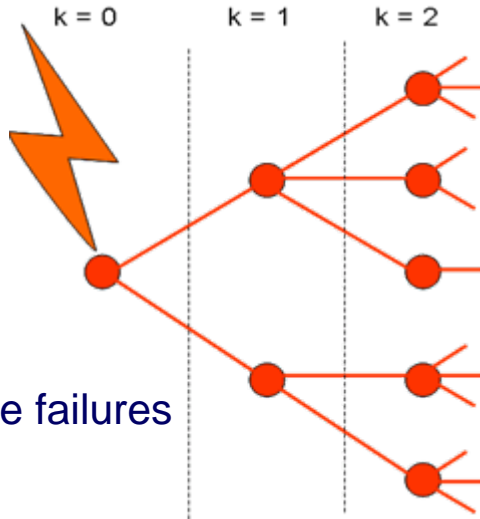
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Identification of interaction networks

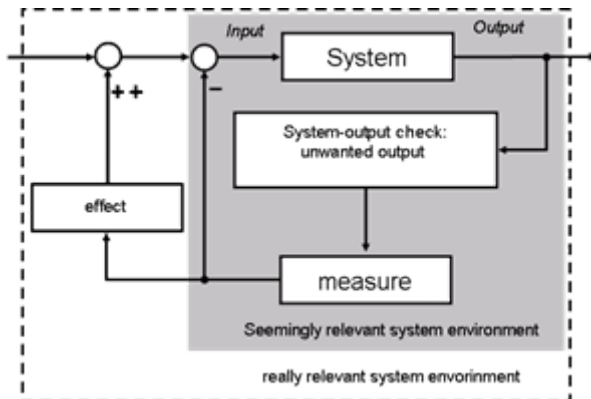
Example: blackout



Disaster dynamics: What are we interested in.

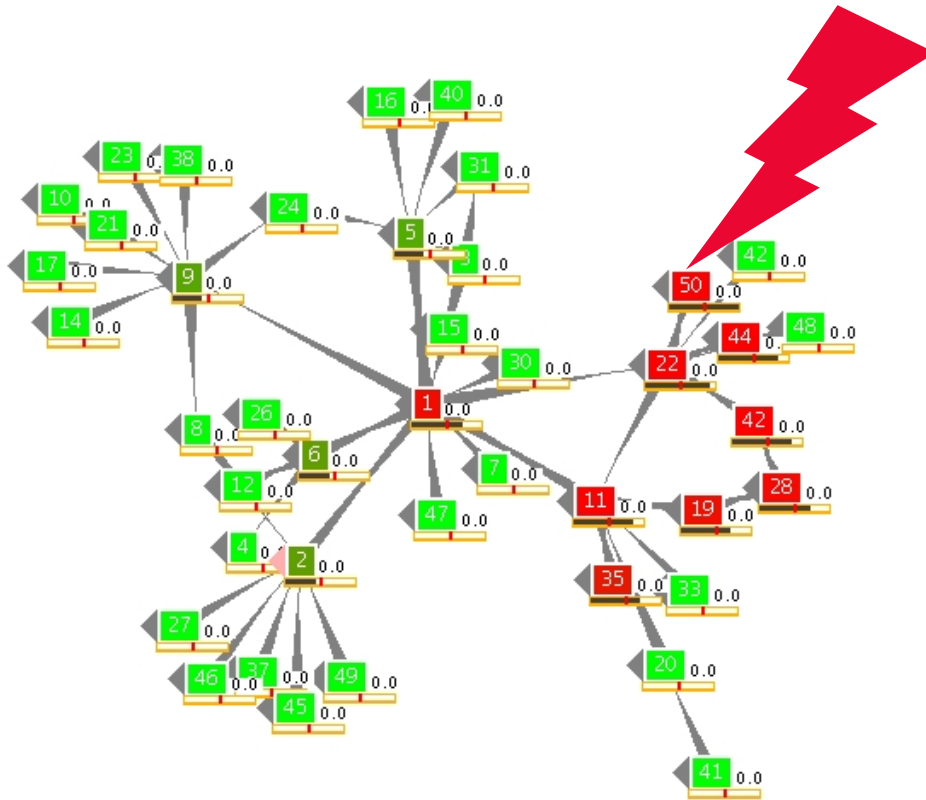


Cascade-like failures



Vicious cycles





Spreading of disasters:

- Causal dependencies (directed)
- Initial event (internal, external)
- Redistribution of loads
- Delays in propagation
- Capacities of nodes (robustness)
- Cascade of failures

Simulation of topology dependent spreading:

- What are the influences of different network topologies and system parameters?
- Optimal recovery strategies?

Buzna L., Peters K., Helbing D., Modelling the Dynamics of Disaster Spreading in Networks, Physica A, 2006

Node model

Node dynamics:

$$\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta \left(\sum_{j \neq i} \frac{M_{ij} x_j(t - t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau} \right) + \xi_i(t)$$

x_i - state of the node

$x_i = 0$ state: usual situation

$x_i > \theta_i$ state: node is destroyed

θ_i - node threshold

t_{ij} - time delay

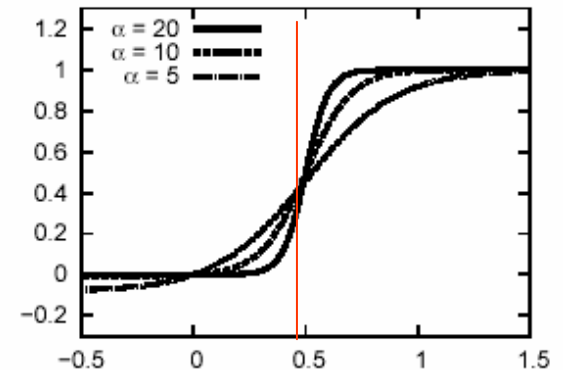
M_{ij} - link strength

a, b, α, β - fit parameters

$\xi_i(t)$ - internal noise

O_i - node out-degree

Threshold function:



$$\Theta(x) = \frac{1 - \exp(-\alpha x)}{1 + \exp[-\alpha(x - \theta_i)]}$$

Node degree:

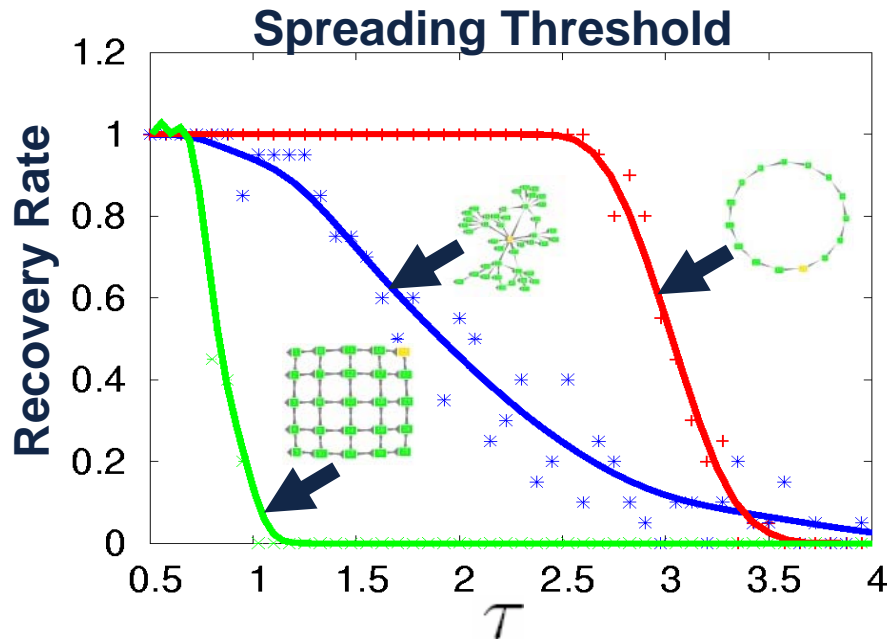
$$f(O_i) = \frac{aO_i}{1 + bO_i}$$



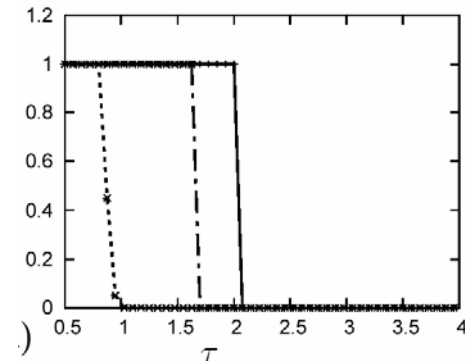
We use a directed network, dynamical, bistable node models and delayed interactions along links.

Node robustness vs. failure propagation:

$$\frac{dx_i}{dt} = \underbrace{-\frac{x_i}{\tau}}_{\text{recovery}} + \Theta \left(\sum_{j \neq i} \frac{M_{ij} x_j (t - t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau} \right) + \xi_i(t)$$



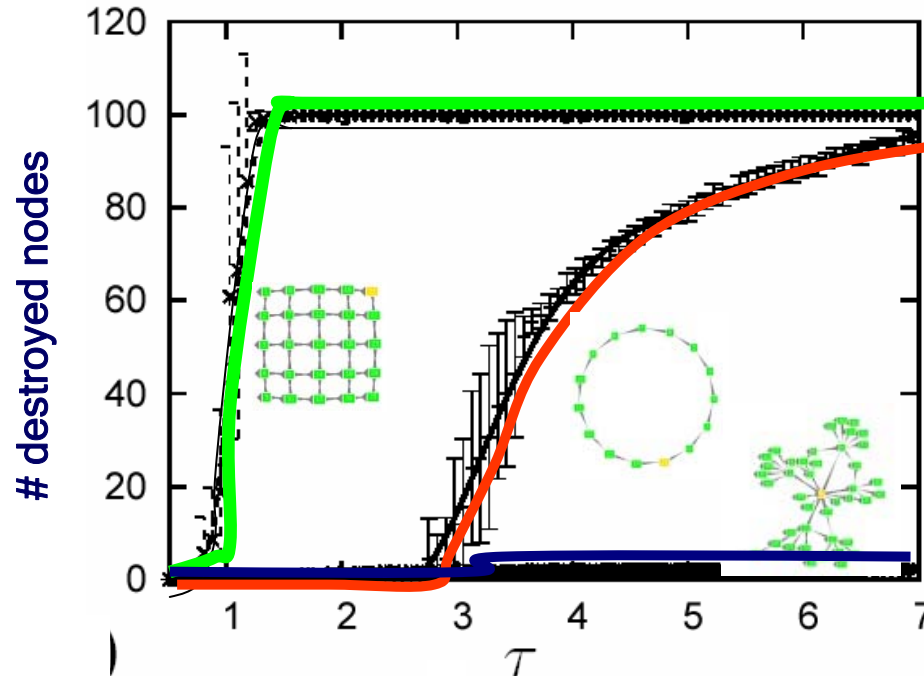
homogeneous networks:



**➔ We found a critical threshold for the spreading of disasters in networks.
Topology and parameters are crucial.**

Topology and spreading dynamics

Example: 100 nodes, average state after $t=300$

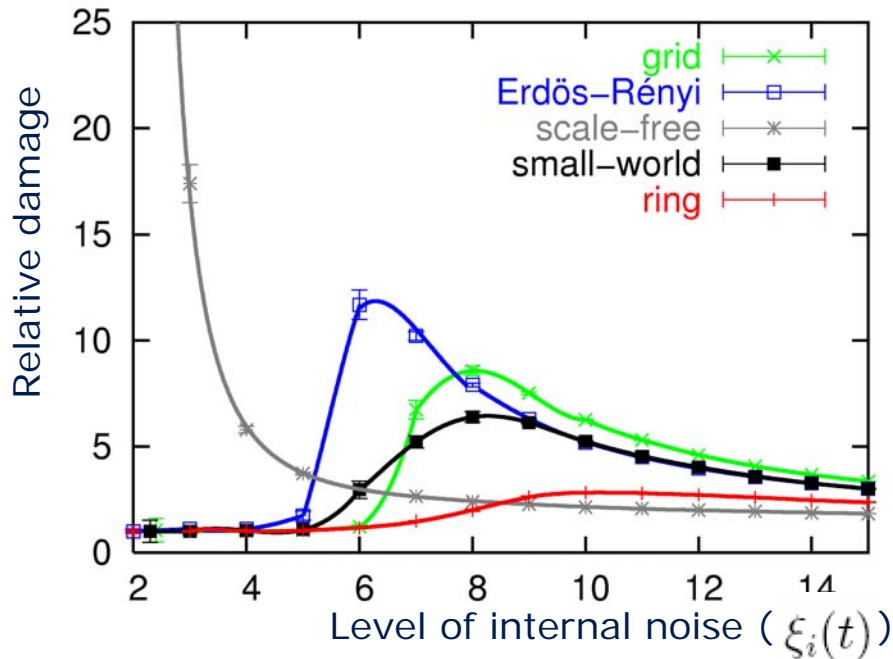


We found a topology dependent „velocity“ of failure propagation. Spreading in scale-free networks is slow.

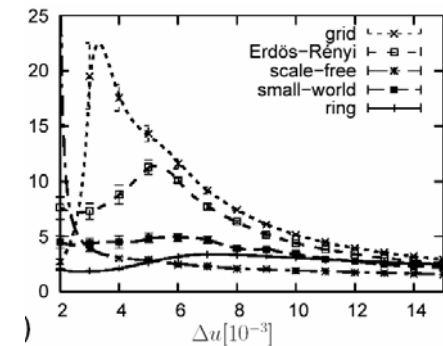
Coinciding, distributed, random failures.

$$\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta \left(\sum_{j \neq i} \frac{M_{ij} x_j (t - t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau} \right) + \xi_i(t)$$

Damage compared to an unconnected "network":



homogeneous networks:



➔ **Connectivity is an important factor (in a certain region).**

Modelling the recovery

1. Mobilization of external resources:

$$r(t) = a_1 t^{b_1} e^{-c_1 t}$$

2. Formulation of recovery strategies

- Network topology
- Level of damage

3. Application of resources

in nodes

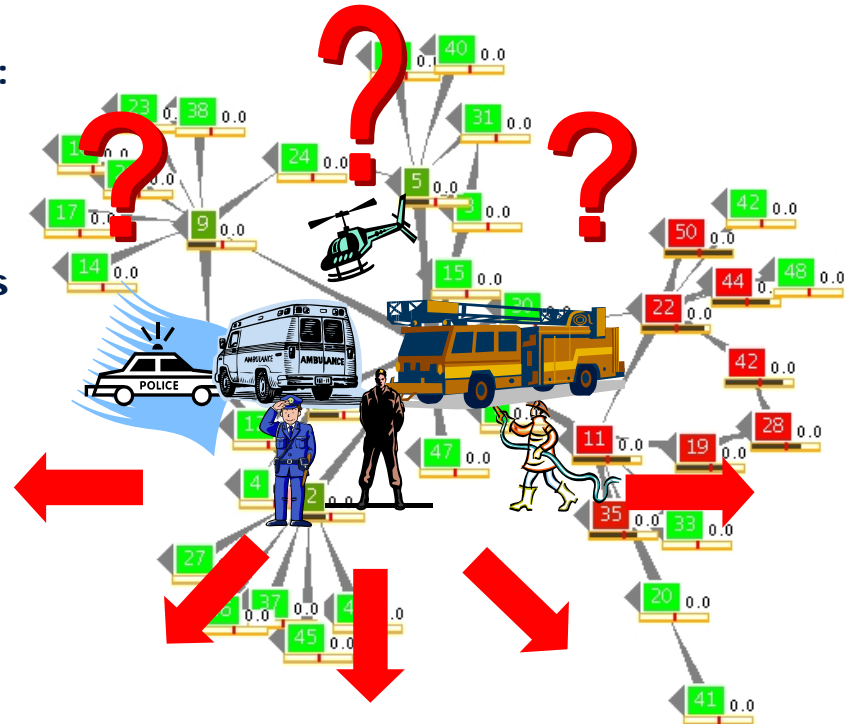
$$\frac{1}{\tau_i(t)} = \frac{1}{(\tau_{start} - \beta_2)e^{-\alpha_2 R_i(t)} + \beta_2}$$

Parameters

Network topology

t_D time delay in response

R disposition of resources



$R_i(t)$ - cumulative number of resources deployed at node i

τ_{start} - initial intensity of recovery process

$\alpha_2 \beta_2$ - fit parameters

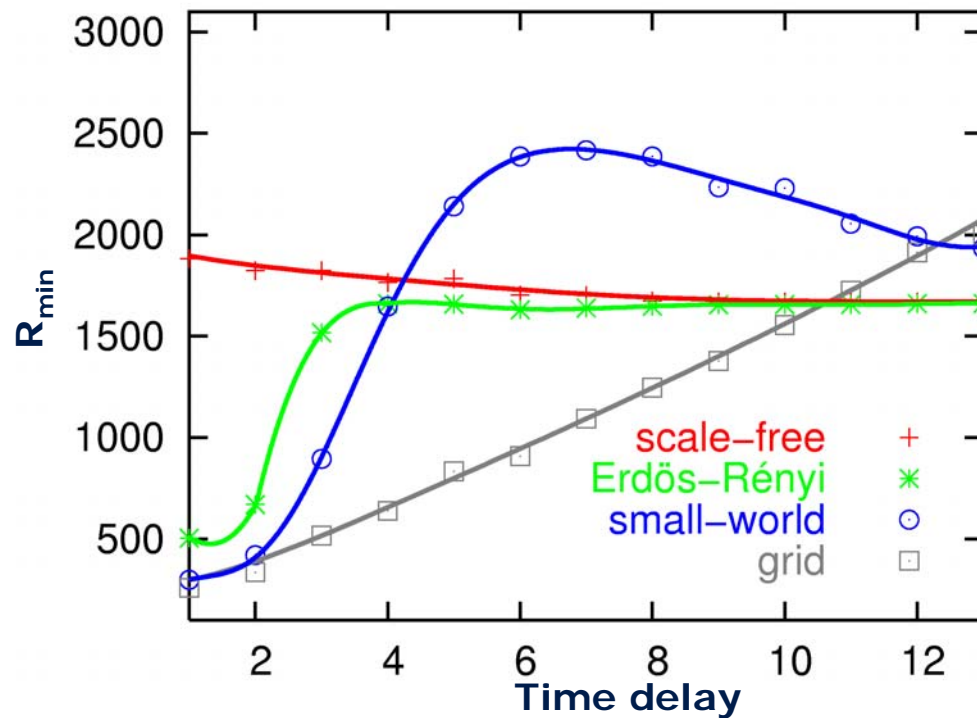
Recovery of networks

Given: amount of resources, mobilized with certain delay.

Is the network able to recover?

Recovery strategy:
uniform protection of all
damaged nodes ($x_i > 0$)

Worst – case scenario



➔ Recovery (in reasonable time) is not always possible.

Formulation of recovery strategies:

Network topology
 Level of damage

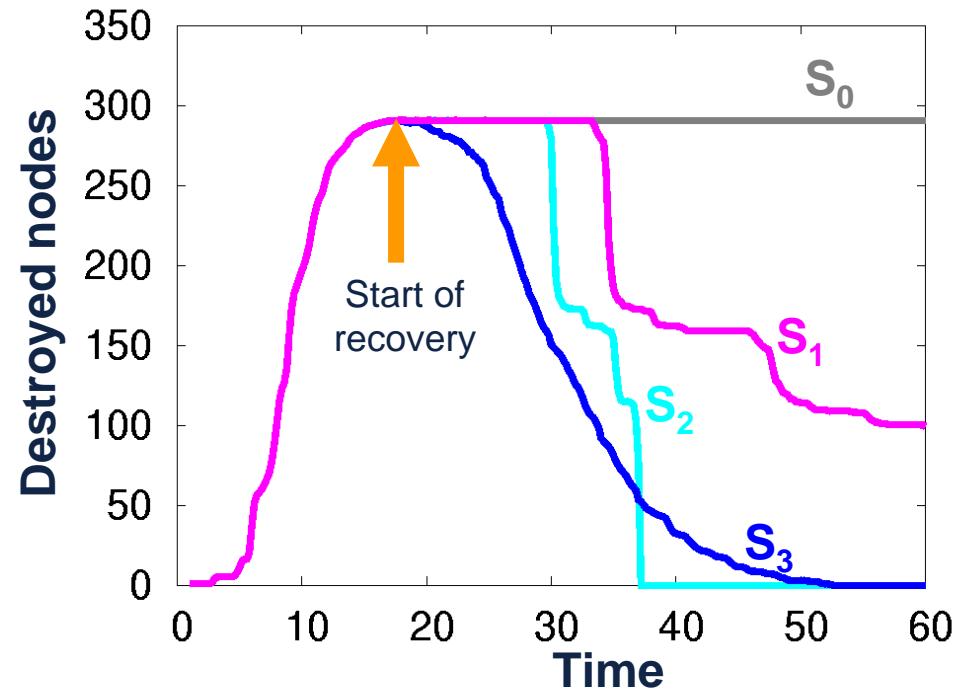
S_0 – no recovery

S_1 – uniform deployment

S_2 – priority1: destroyed nodes
 priority2: damaged nodes

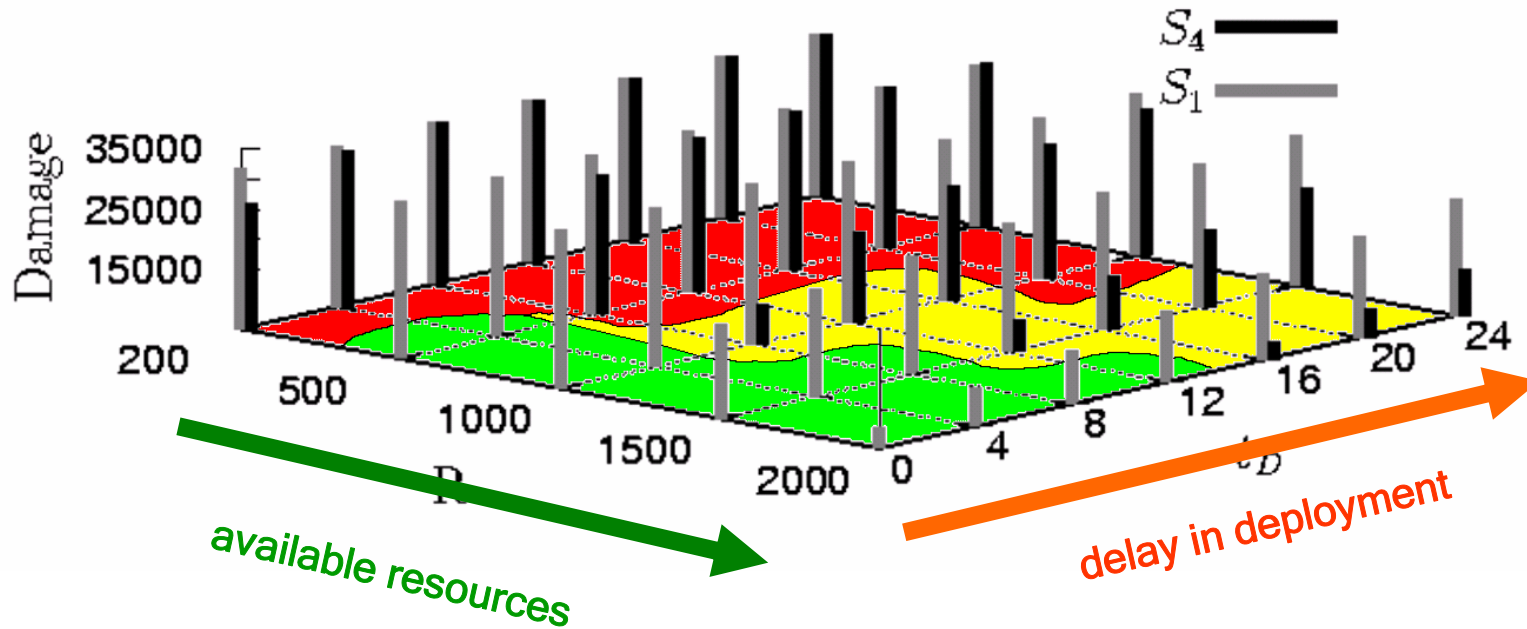
S_3 – out-degree based deployment

Application of resources on a scale-free network



When does strategy matter?

Comparison of efficient and inefficient strategies:



- ➔ The delay of recovery activities is crucial.
- ➔ Optimization of recovery strategies is promising in certain parameter regions.

Average behaviour of strategies

- Strategies based on the network structure are important for scale-free structures.
- Strategies based on damage information are more appropriate for regular networks.
- The optimal strategy is time dependent!
(short $t_D \Rightarrow$ damage)
(large $t_D \Rightarrow$ network structure)

- We proposed a generic model for the spreading of failures in dynamic networked systems.
- The model facilitates an assessment of the stability and robustness of interaction networks and infrastructures.
- It assists the evaluation of disaster response management strategies.

Topology aspects:

- Phase transition in dynamic behaviour
- Different spreading conditions
- Robustness under distributed random failures

There is no unique robust and reliable architecture !
e.g.: redundancy, hubs, feedback loops

Recovery aspects:

- Effectiveness of damage oriented or connectivity dependent response strategies
- Minimum of resources to stop an evolving disaster
- Optimization of disaster response

There is no unique optimal response strategy!
e.g.: delay, available forces, topology

Thank you for your attention.

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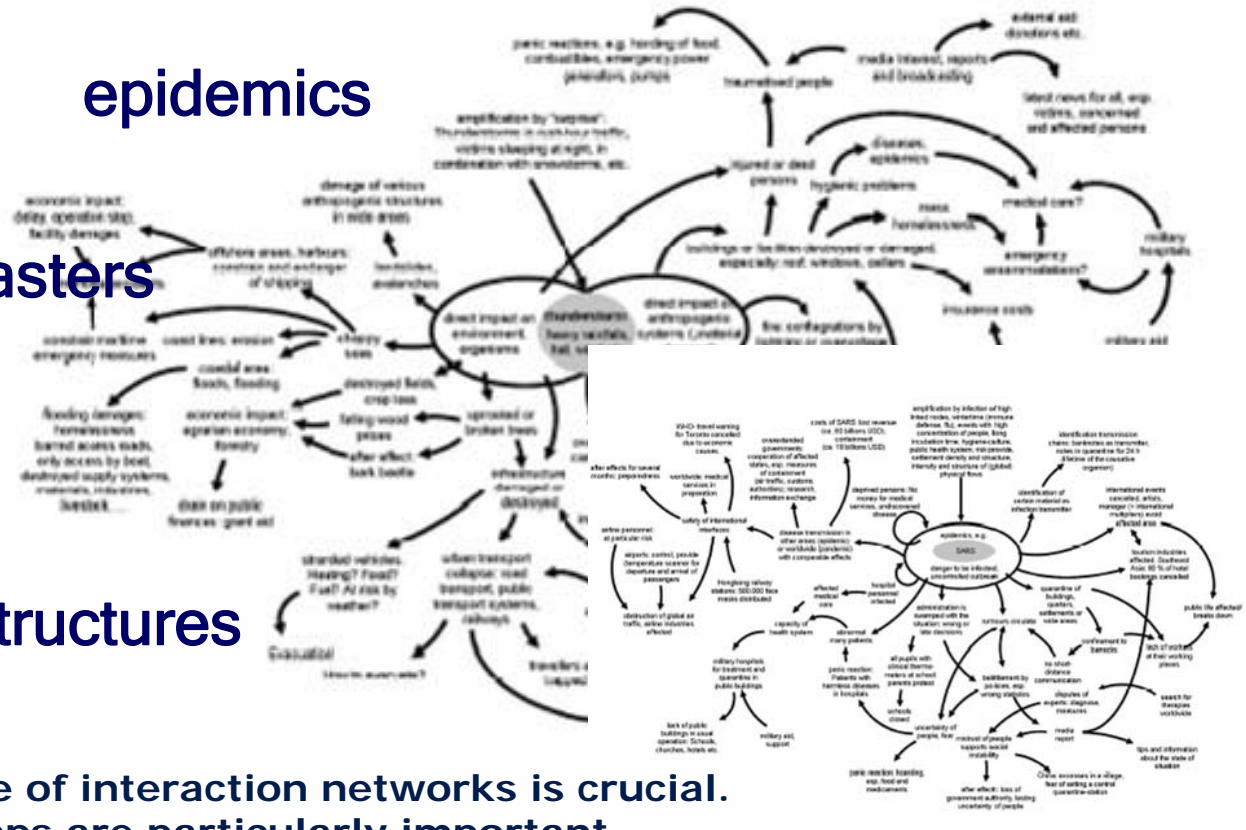
blackouts

epidemics

natural disasters

- Sanstorms
- Hurricanes
- Floods
- earthquakes

infrastructures

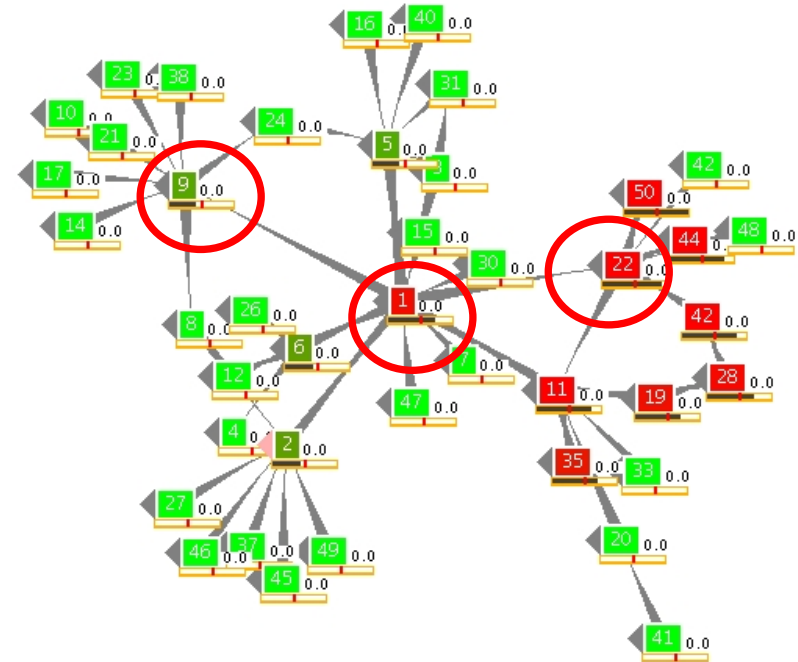


The structure of interaction networks is crucial. Feedback loops are particularly important.

D. Helbing, H. Ammoser, C. Kühnert: Disasters as extreme events and the importance of networks for disaster response management, Springer (Berlin 2006) (in print)

Importance of hubs in networks ?

- Inhomogeneities can have considerable damping effects on spreading failures
- Hubs reduce the robustness against (small) disturbances and attacks.
- Scale-free networks are among the safest structures in case of large and distributed failures.




Hubs play a quite ambiguous role !