

# PUBLIC TRANSPORT NETWORKS: SCALING AND VULNERABILITY

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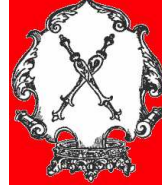
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## Motivation

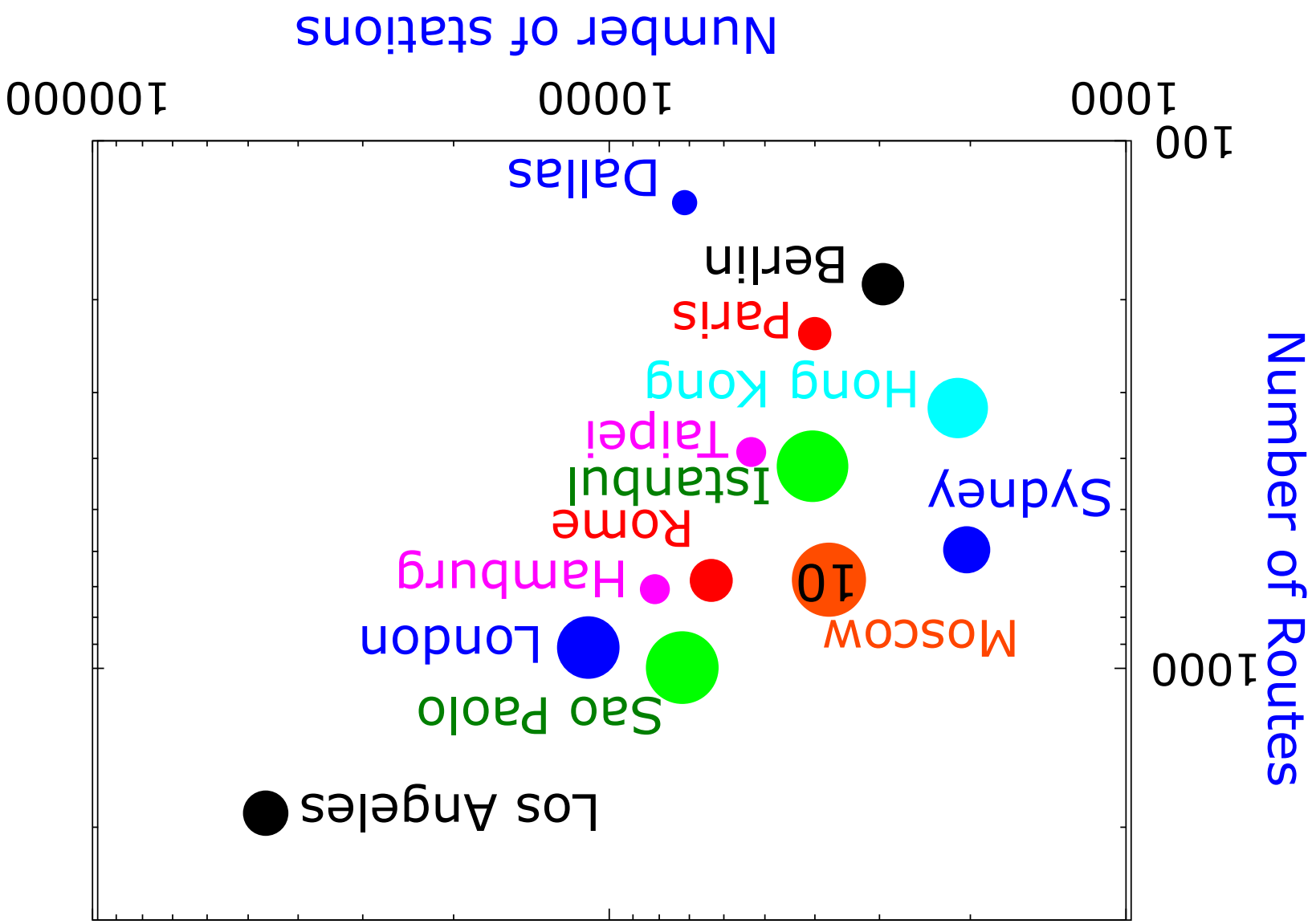
- Analysis of public transport (PT) networks of **various means of transport**  
cf. Boston subway (Marichiori, Latora'00-'02), Vienna U-Bahn (Seaton, Hackett'04)

- Study of PT networks on a **larger database**

cf. 22 cities in Poland, Warsaw:  $N = 1530$  (Sienkiewicz, Høyst'05), Berlin, Düsseldorf, Paris (von Ferber et al.'05)

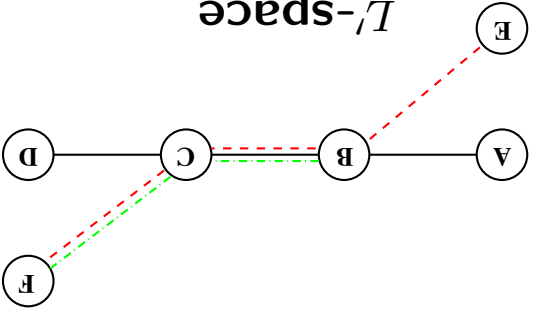
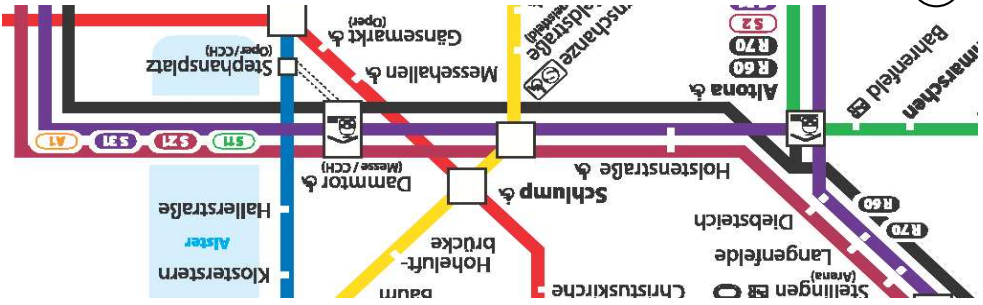
- Study of **specific phenomena** on PT networks or of their **specific features**  
'harness effect', vulnerability

- **Computer simulations** of PT networks

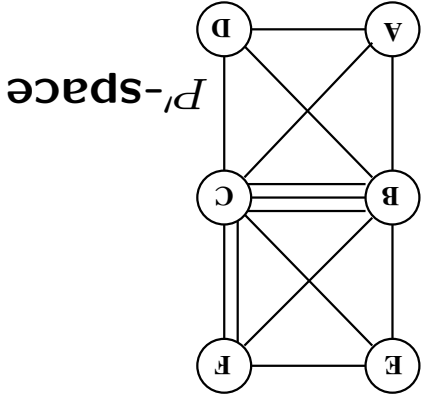


Database

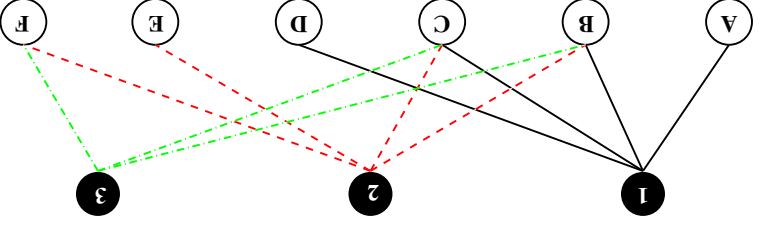
# Network Interpretation



$L'$ -space

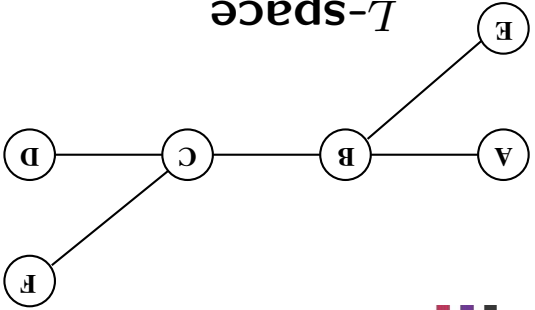


$P'$ -space

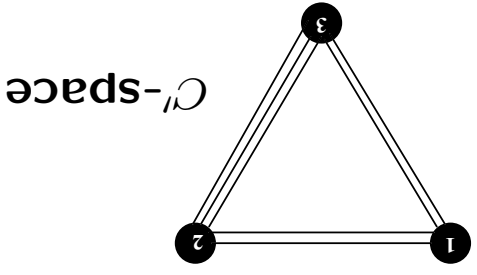


(Bipartite)  $B$ -space

*Sienkiewicz, Hołyst'05*

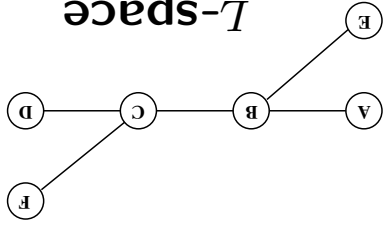


$L$ -space

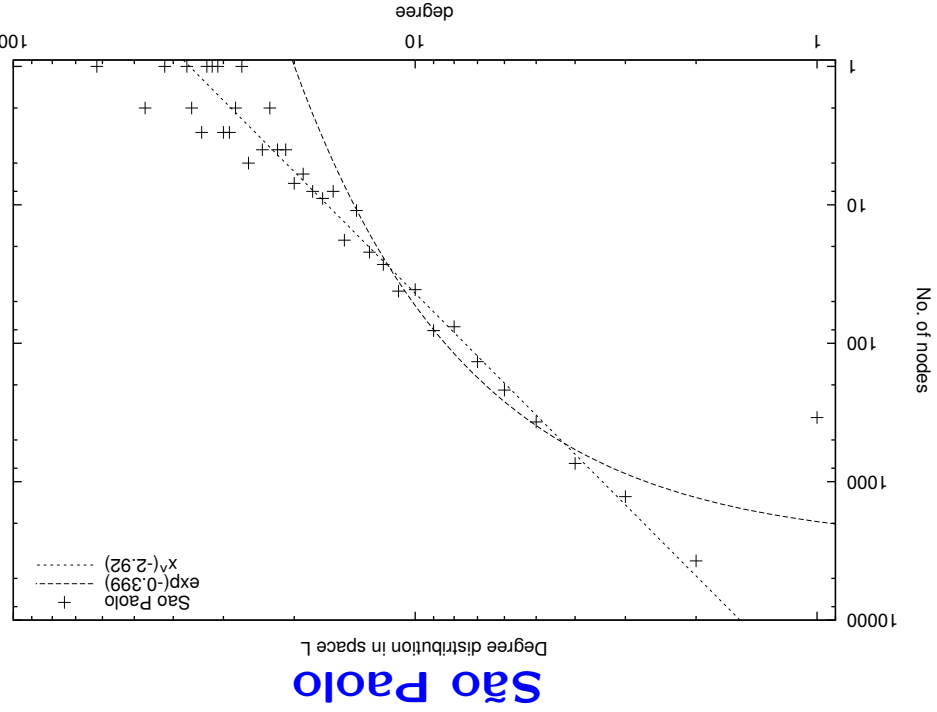
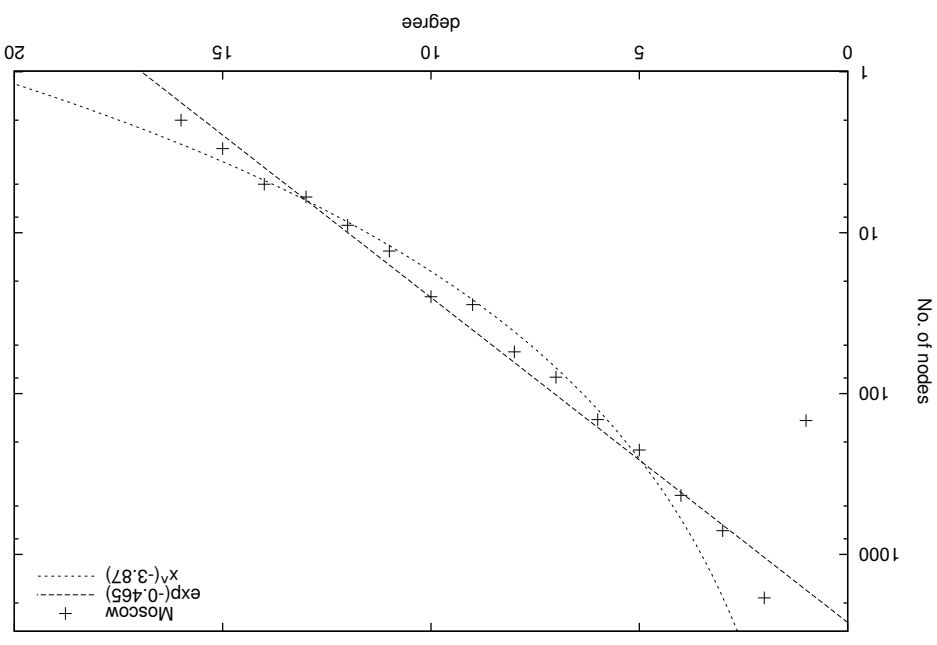


$C'$ -space

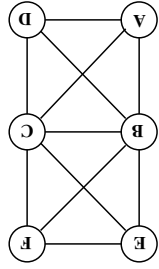
# Scale-free behaviour in PT networks?



**Node degree distribution:**

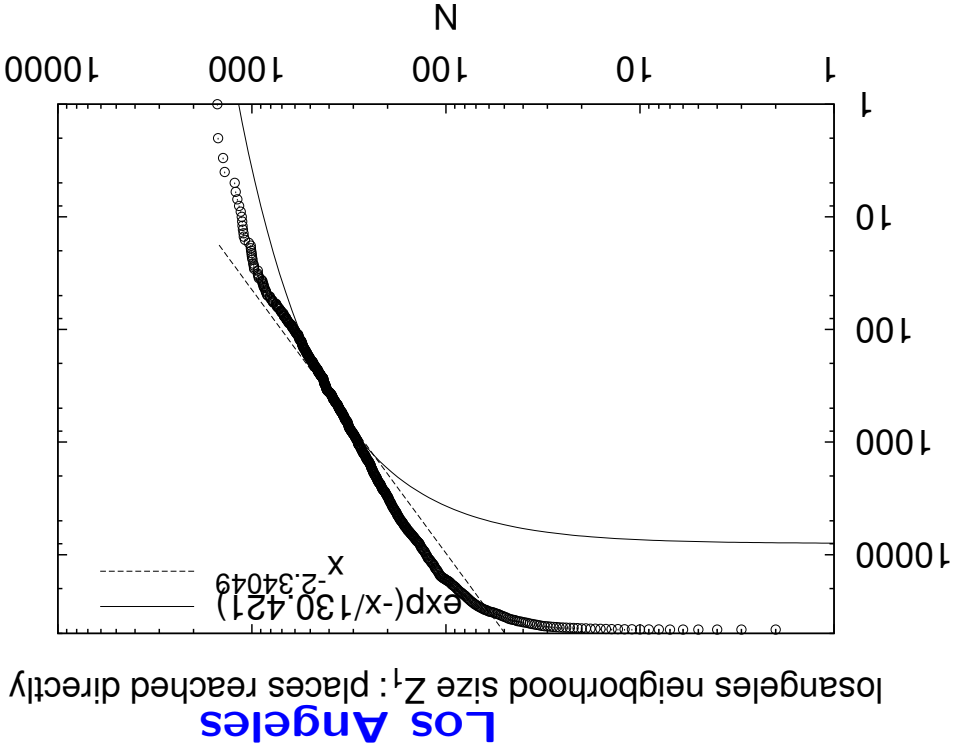
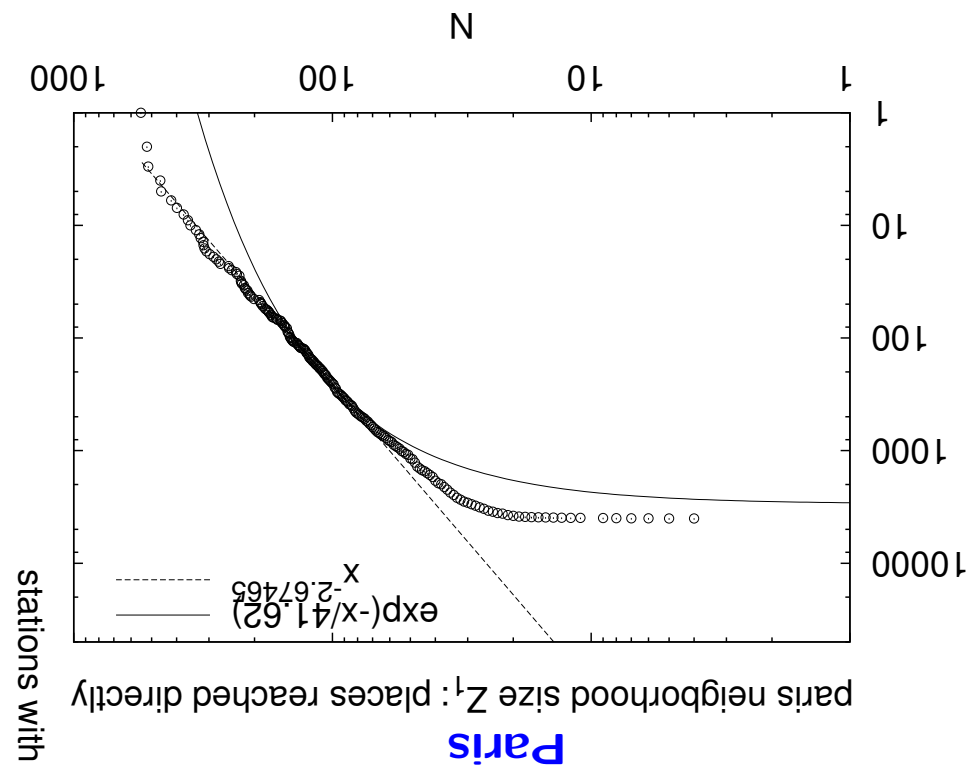


# Scale-free behaviour in PT networks?



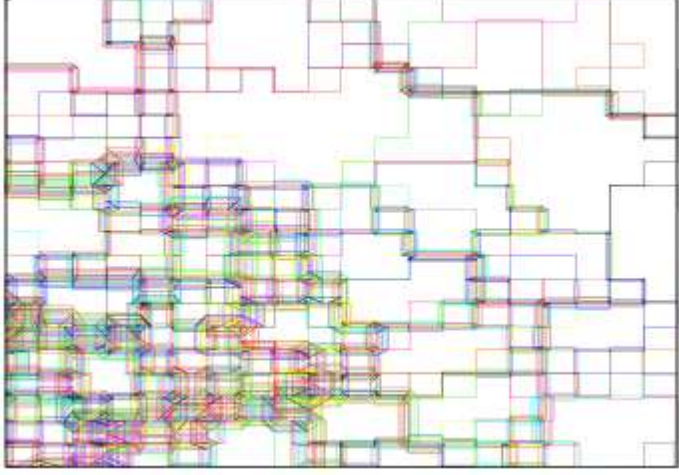
P-space

stations with  $Z_1 \geq N$



# Computer simulations of PT networks

Simulated network,  $a = 0, b = 0.5$

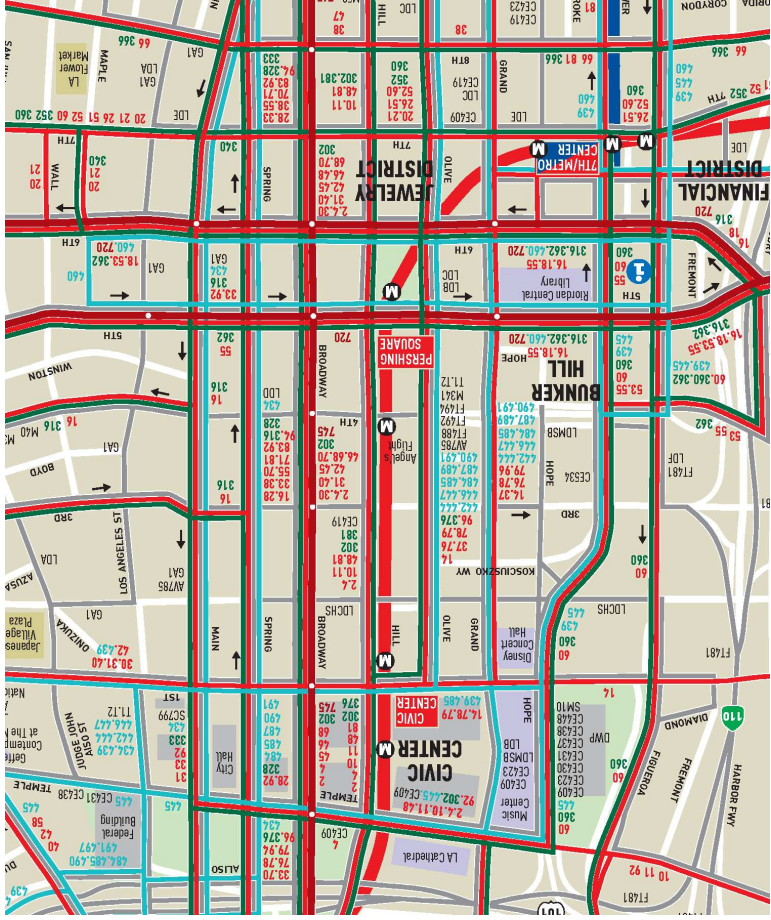


1. First route is a self-avoiding walk.
2. Add a route:

- 2a) choose the first site with preference  $d \sim k(r) + a$
- 2b) choose the next site with preference  $d \sim k(r) + b$

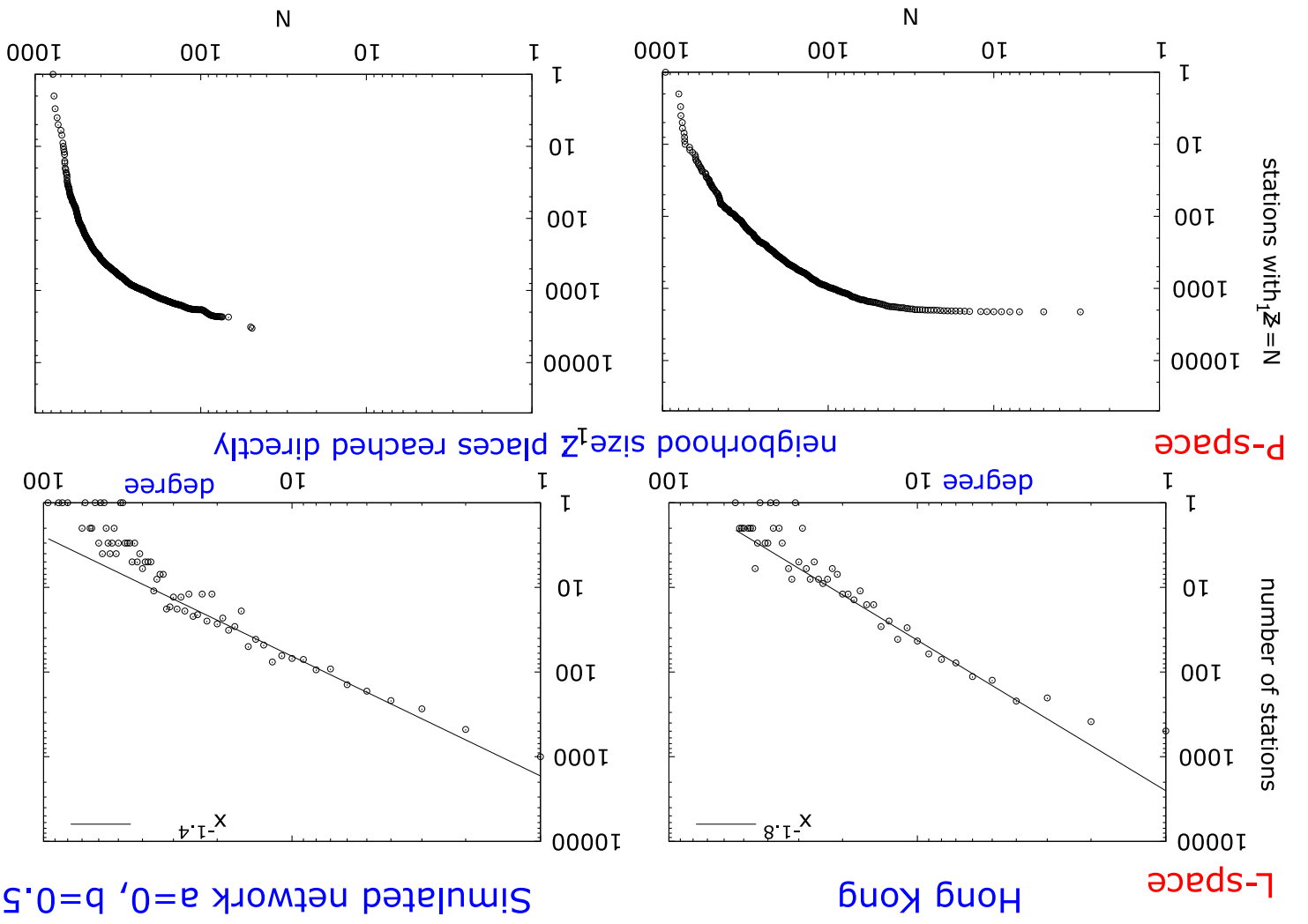
repeat, discard at self-intersection

Downtown Los Angeles



$k(r)$ : degree of site  $r$

# Computer simulations of PT networks

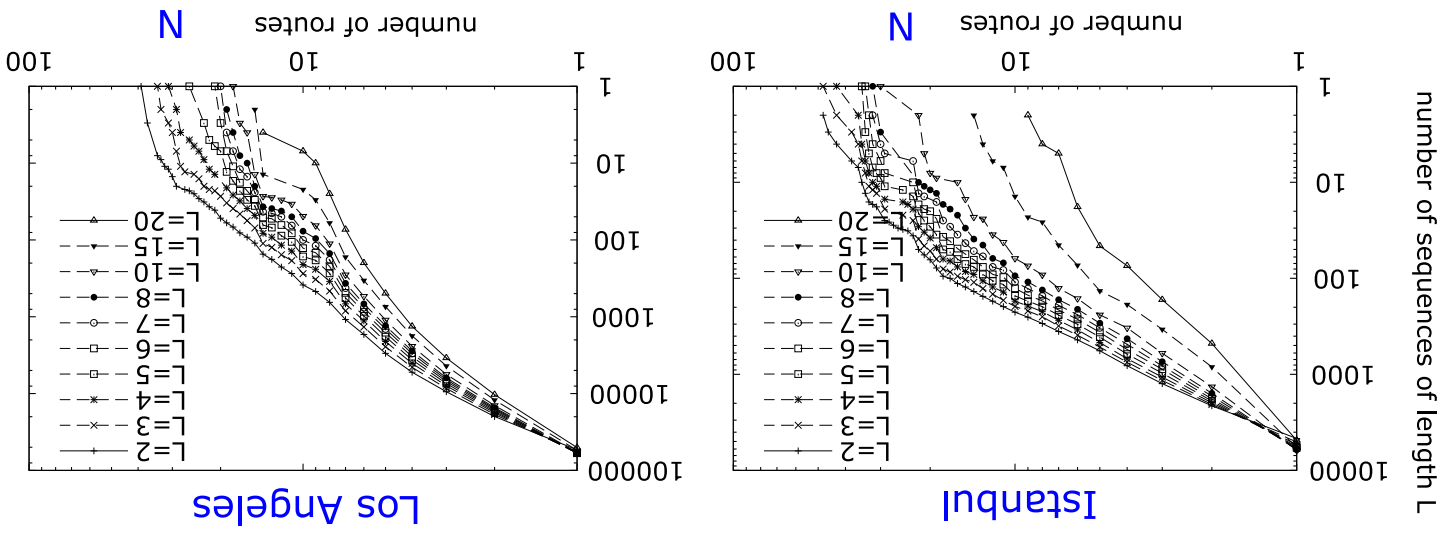




# Harness effect

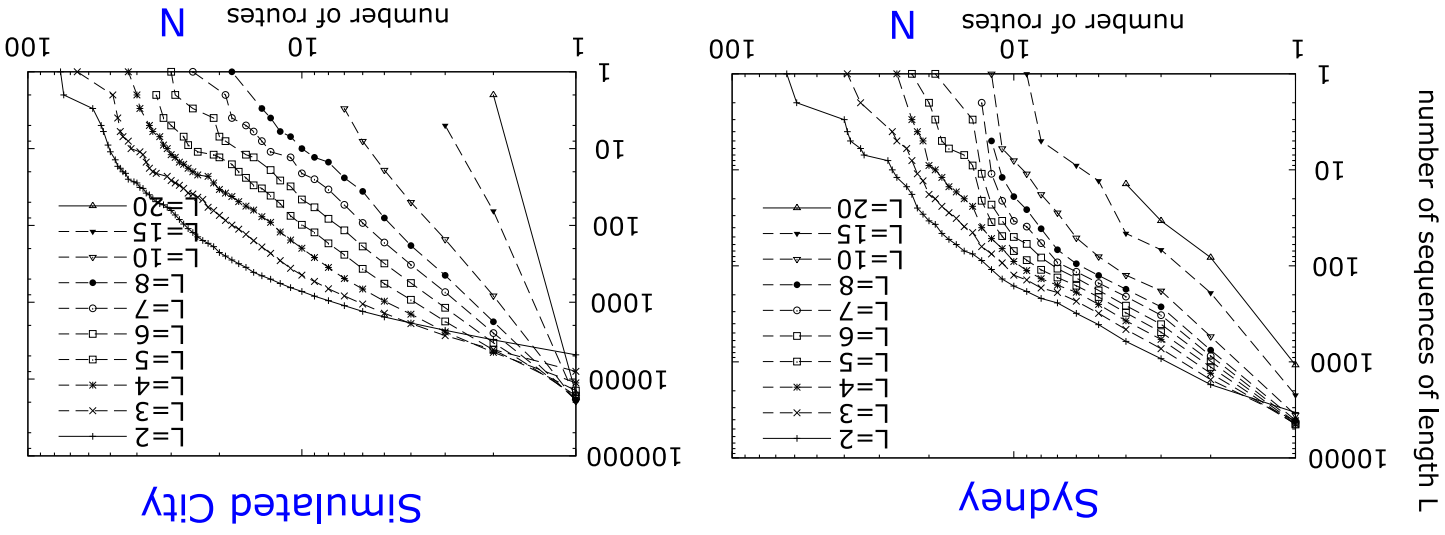


Routes share common roads/rails for a number of stations.  
 Distribution  $P(N, L)$ :  $N$  routes on segment of length  $L$ .

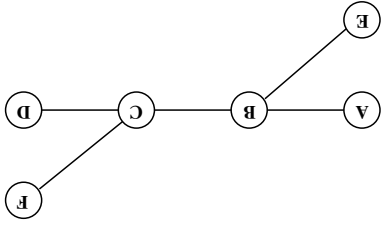


# Harness effect

Distribution  $P(N, L)$ :  $N$  routes on segment of length  $L$ .

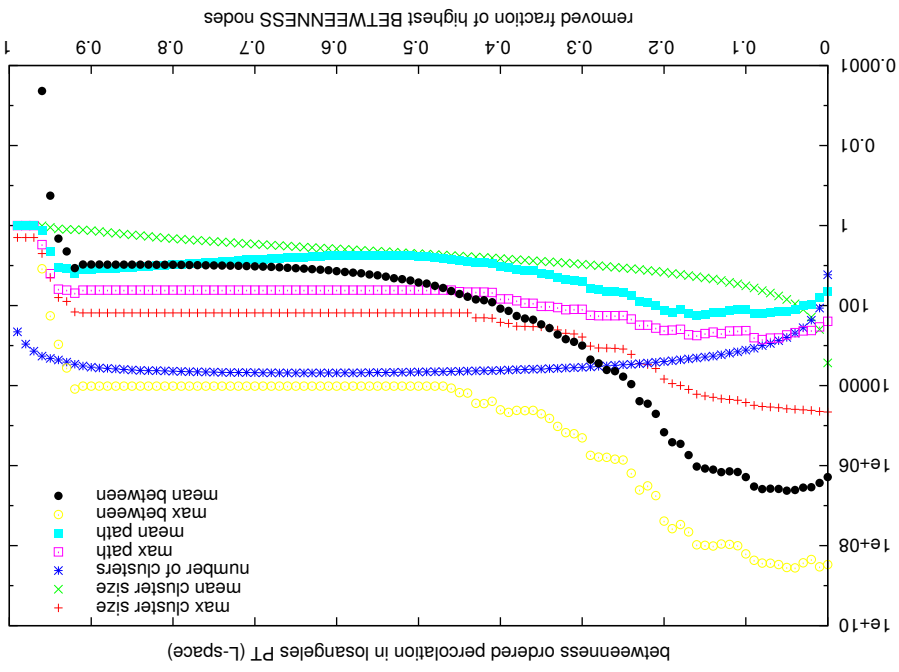


# Vulnerability of PT networks

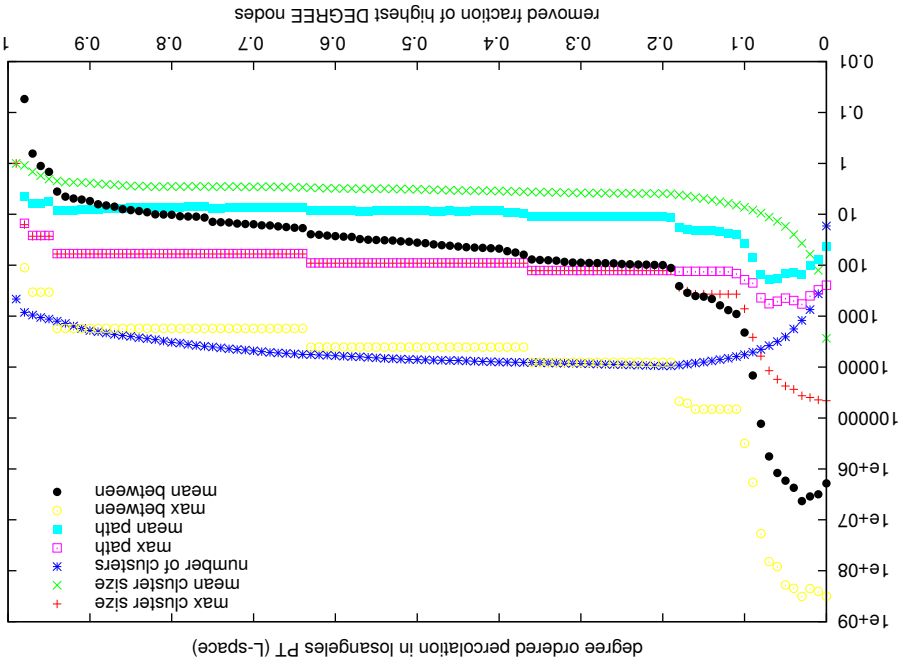


*L*-space

## Highest betweenness vulnerability



## Highest degree vulnerability



## Conclusions

- Small worlds (small  $< l >$ , high  $< C >$ )
- Scale-free (very often in  $L$ -space, more seldom in  $P$ -space)
- Specific features (Harness effect, Vulnerability)
- Reproducible by the evolutionary model