# Multi-asset minority games

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1. Minority Games with **one** asset :
   a. Economics : competition under uncertainty, inductive reasoning
   b. Physics : phase transition, anomalous fluctuations
   c. Mathematics : exact solution
2. Minority Games with **many** assets :
   a. how do speculators distribute their trading volume depending on the information content of the different assets?
   b. how do incentives to trade affect the composition of the portfolios?
   c. how does speculative trading "dress" financial correlations?
   d. phase structure?

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# Minority game basics

a. Traders react to the receipt of public information $\mu(t)$ by formulating a simple binary bid (buy/sell) $b_i(t) \in \{-1, 1\}$

b. They receive the payoff $-b_i(t)A(t)$ , $A(t) = \sum_i b_i(t)$

c. They have fixed prescribed decision schemes ("trading strategies")

d. Agents are inductive : they keep track of the performance of each of their strategies and use at each time step the one that performed better in the past

$$r(t) \equiv \log p(t) - \log p(t-1) \propto A(t)$$

**Information** : exogenous/endogenous

$$\mu(t) \in \{1, \ldots, P\} \ , \ \ \log P \simeq \text{memory of agents}$$

**Trading strategies** : quenched disorder/heterogeneity

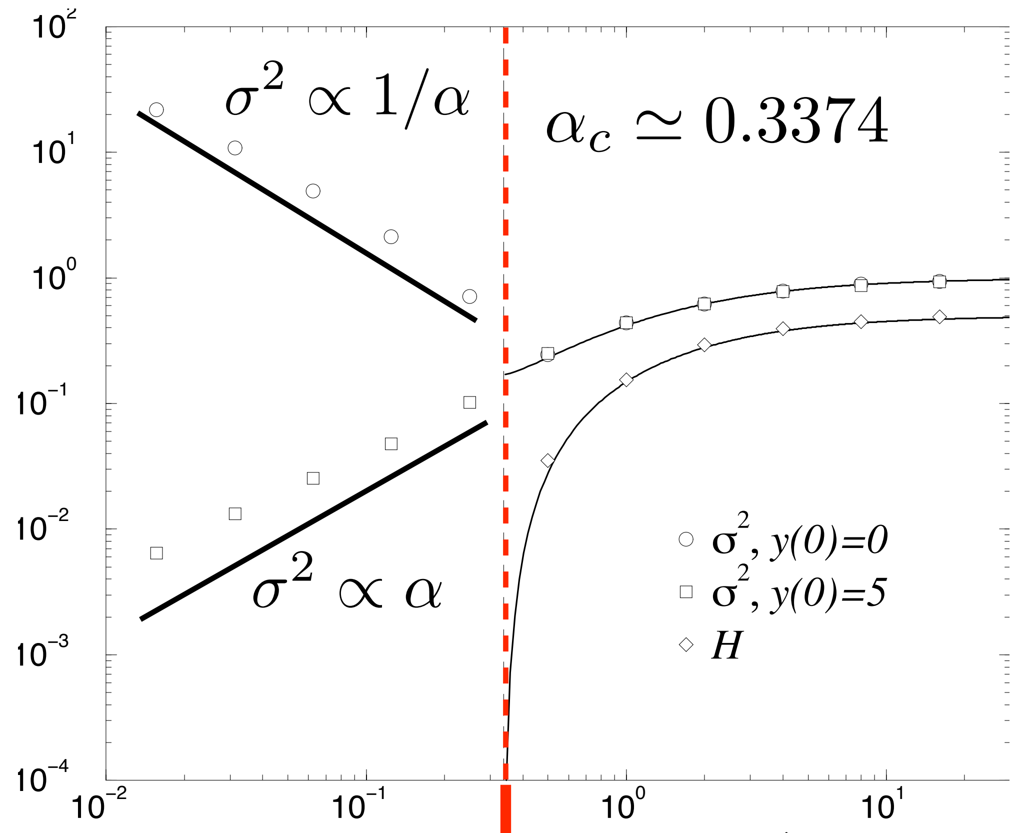
# Phase transition

**Volatility**

$$\sigma^2 = \langle A^2 \rangle$$

**Predictability**

$$H = \frac{1}{P} \sum_{\mu=1}^{P} \langle A|\mu \rangle^2$$




**Theory :**

$$N \to \infty$$

$$P \propto N$$

| $\alpha < \alpha_c$ | $\alpha > \alpha_c$ |
|---|---|
| $\langle A\|\mu \rangle = 0 \ \forall \mu \ \to$ unpredictable | $\exists \mu$ s.t. $\langle A\|\mu \rangle \neq 0 \ \to$ predictable |
| dynamics is not ergodic | dynamics is ergodic |

# Many-assets model

**Trading strategy** : fixed vector $\boldsymbol{a}_{ig} = \{a_{ig}^{\mu}\}$ , $g \in \{1, \ldots, S\}$

which strategy? ➡ $g_i(t) = \arg\max_g U_{ig}(t)$

choice , return ➡ $b_i(t) = a_{i,g_i(t)}^{\mu(t)} \rightarrow A(t) = \sum_i b_i(t)$

learning ➡ $U_{ig}(t+1) - U_{ig}(t) = -a_{ig}^{\mu(t)} A(t)/N$

**Many assets** : each strategy refers to a different asset $\sigma \in \{1, \ldots, S\}$

which asset? ➡ $s_i(t) = \arg\max_\sigma U_{i\sigma}(t)$

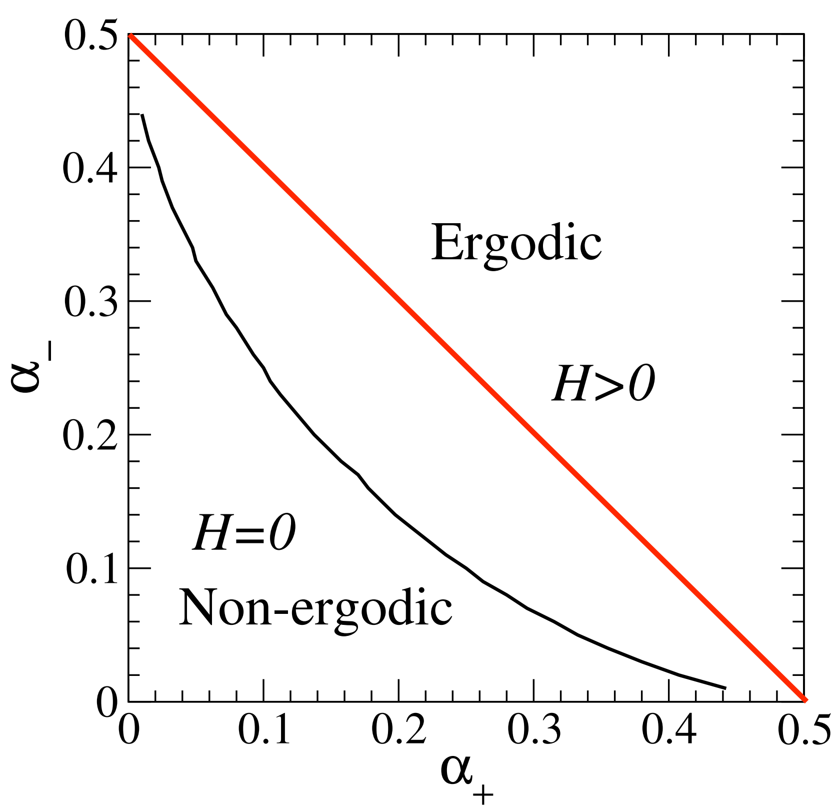
choice , return ➡ $b_i(t) = a_{i,s_i(t)}^{\mu_\sigma(t)} \rightarrow A_\sigma(t) = \sum_i a_{i\sigma}^{\mu_\sigma(t)} \delta_{\sigma, s_i(t)}$

learning ➡ $U_{i\sigma}(t+1) - U_{i\sigma}(t) = -a_{i\sigma}^{\mu(t)} A_\sigma(t)/N$

$\mu_\sigma \in \{1, \ldots, P_\sigma\}$ no correlation $\Rightarrow \langle A_+ A_- \rangle \simeq 0$

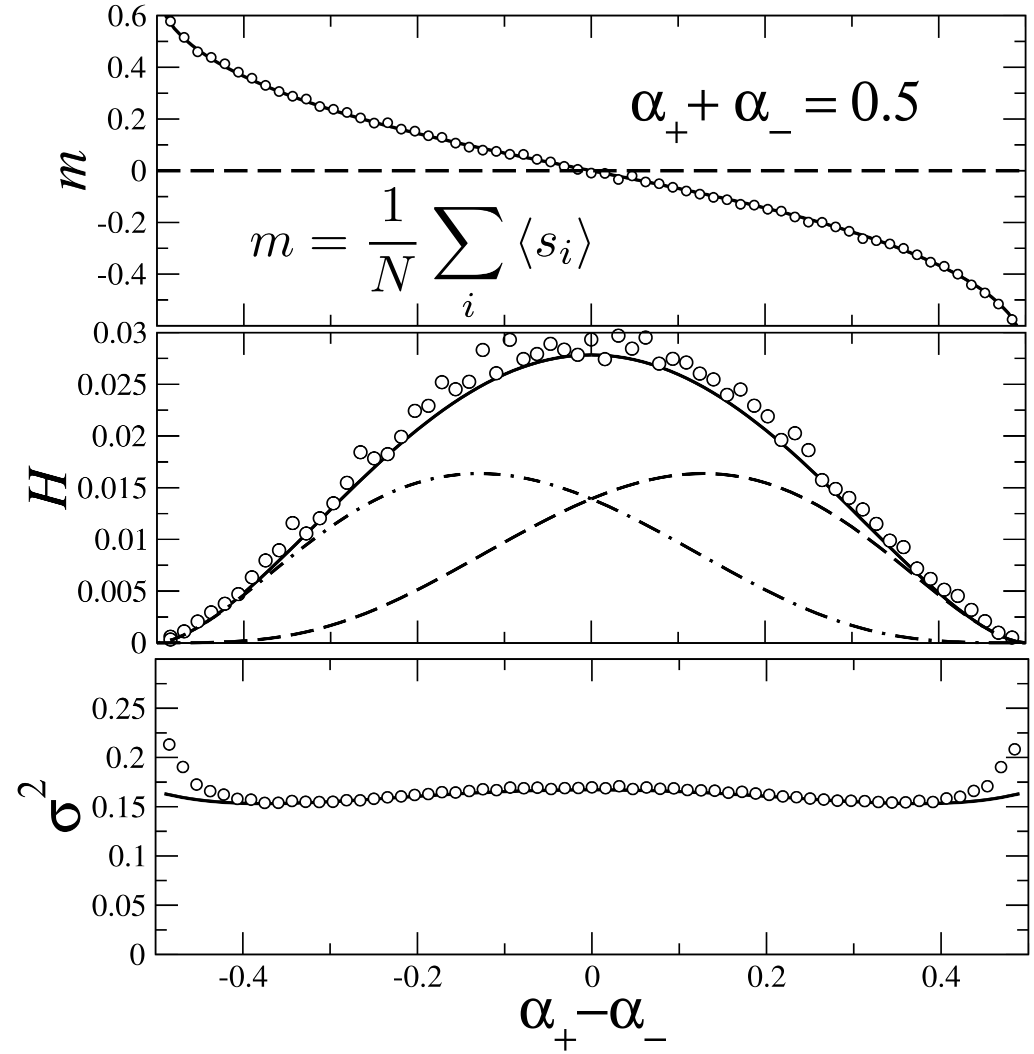
# Two-assets model

Phase diagram




$$\alpha_\sigma = P_\sigma / N \qquad \sigma \in \{-1, 1\}$$

$$H = \sum_\sigma H_\sigma \qquad H_\sigma = \frac{1}{P_\sigma} \sum_{\mu_\sigma} \langle A_\sigma | \mu_\sigma \rangle^2$$




agents are captured by information-poor markets!

# Two-assets model : grand-canonical

Two types of traders :

**Speculators** have incentives to trade and may abstain

**Producers** always trade (provide extra information)

$$N = P_\sigma / \alpha_\sigma \text{ speculators} \ , \ N_p^\sigma = n_p P_\sigma \text{ producers}$$

## Dynamics of speculators

which asset? ➡ $$s_i(t) = \arg\max_\sigma U_{i\sigma}(t)$$

choice , return ➡ $$\phi_{i\sigma}(t) = \delta_{s_i(t),\sigma}\theta[U_{i\sigma}(t)] \ \rightarrow \ A_\sigma(t) = B_\sigma(t) + \sum_i a_{i\sigma}^{\mu_\sigma(t)}\phi_{i\sigma}(t)$$

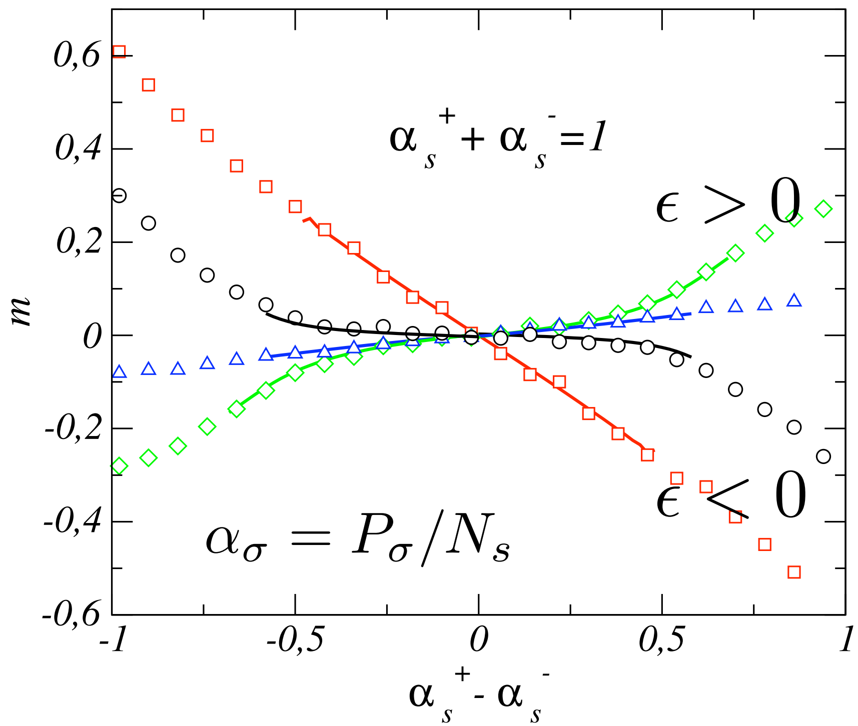
learning ➡ $$U_{i\sigma}(t+1) - U_{i\sigma}(t) = -a_{i\sigma}^{\mu_\sigma(t)} A_\sigma(t) - \epsilon_i$$

$\epsilon_i > 0$ positive incentive (“risk prone”)

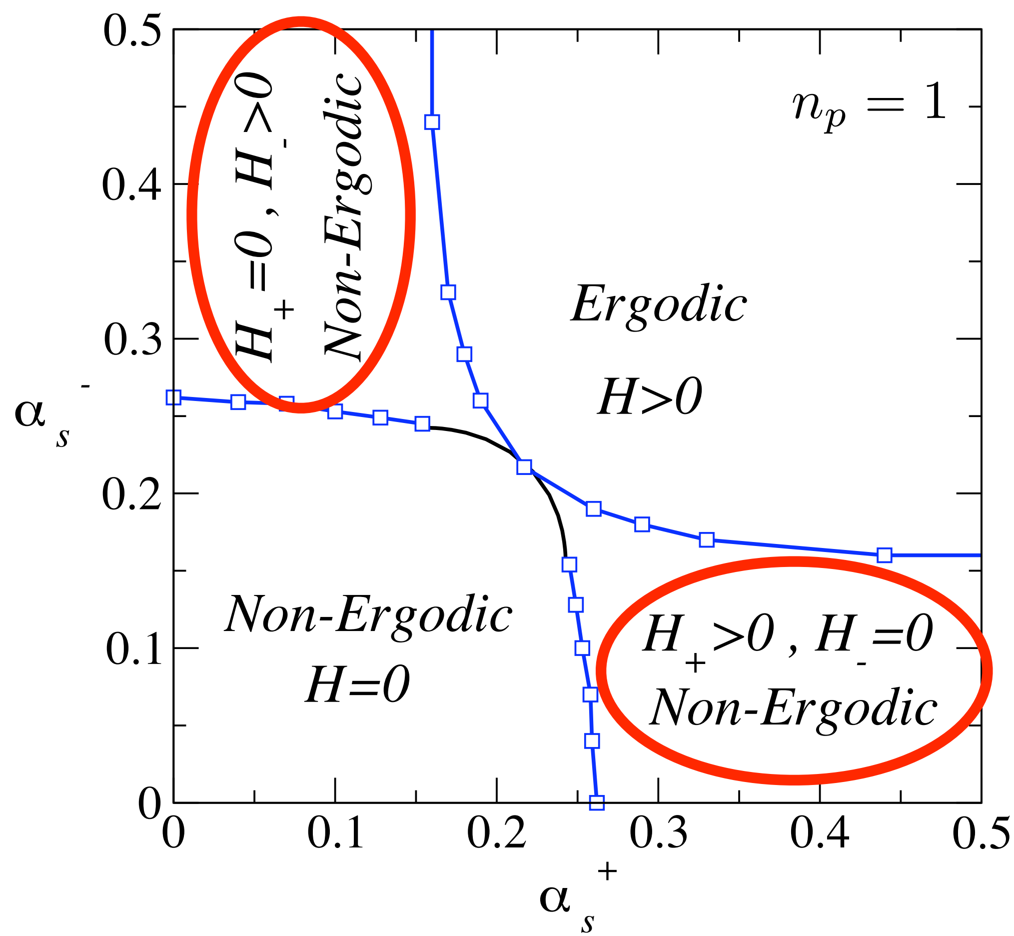
$\epsilon_i < 0$ negative incentive (“risk averse”)

# Results

$$m = \frac{1}{N_s} \sum_{\text{spec.}} (\langle \phi_{i,+} \rangle - \langle \phi_{i,-} \rangle)$$




Phase diagram




$\epsilon < 0$ : same as before

$\epsilon > 0$ : improvement!

# Outlook

- Speculative trading does not contribute sensibly to financial correlations
  - *This may change when agents take risk into account (low-frequency strategies)*
- When there are positive incentives to trade, speculators invest preferentially in the asset with the smallest information content
  - *This is due to the fact that if speculators are forced to trade they contribute to information asymmetries*
- The situation changes when speculators have no incentive to trade, other than making a profit
- Theory : static and dynamical solutions
- Open : Interacting markets? Multiple signals?

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