Bose-Einstein Condensation of Financial (Profit Seeking) Bosons

Kęstutis Staliūnas

Universitat Politècnica de Catalunya (UPC);
Institució Catalana de Recerca i Estudis Avançats (ICREA);

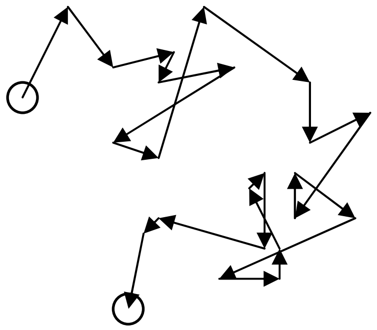
- *Fat Tails (Nongaussian Distributions) in Financial Data;*
- *Fat Tails in distribution of Data-Makers;*
- *Bose-Einstein Condensation;*
- *Thermodynamics of Profit Seeking Bosons;*

Econophysics

- Autocorrelations;
- (Nongaussian) Distributions;
- Crises, financial bubbles;

L.Bachelier 1900.

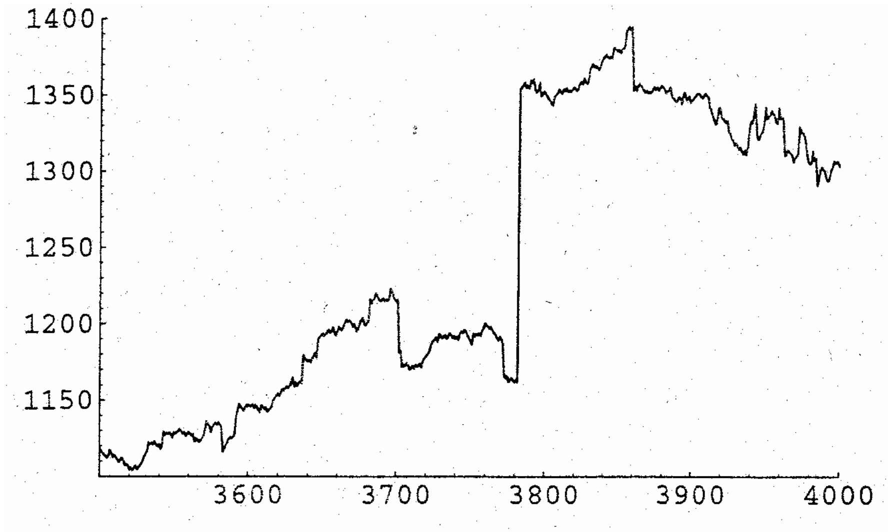
Price diffusion =
Random Brownian walk!



$$P(\Delta X, \Delta t) = \frac{1}{\sqrt{2\pi}\sigma(\Delta t)} \cdot \exp\left(\frac{-\Delta X^2}{2\sigma^2(\Delta t)}\right)$$

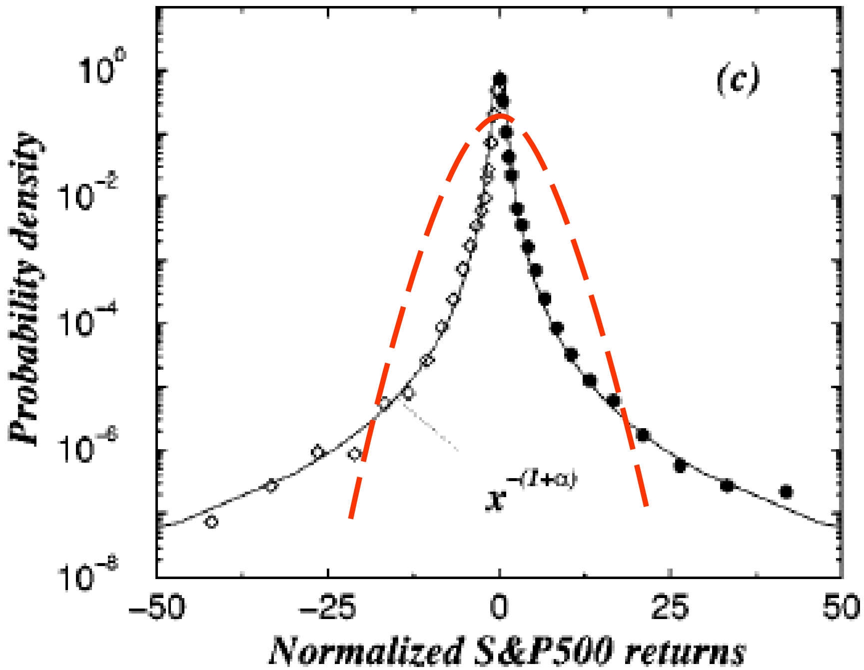
$$\sigma(\Delta t) \propto \Delta t^{1/2}$$

Nongaussian



B.Mandelbrot;

~~$P(\Delta X) = \exp\left(-\Delta X^2\right)$~~



$$P(\Delta X) \propto \frac{1}{|\Delta X|^{\alpha+1}}$$

$1 < \alpha < 2$

$\alpha \approx 1.5$

P.Levy;

$$P(\Delta X) \propto \begin{cases} |\Delta X|^{-(1+\alpha)} & \alpha \approx 3 \quad ? \\ \exp(-\Delta X/\sigma) & ? \end{cases}$$

$\Delta X >> \sigma$

why nongaussian?

Why nongaussian

Nongaussian price changes – nongaussian price changers!

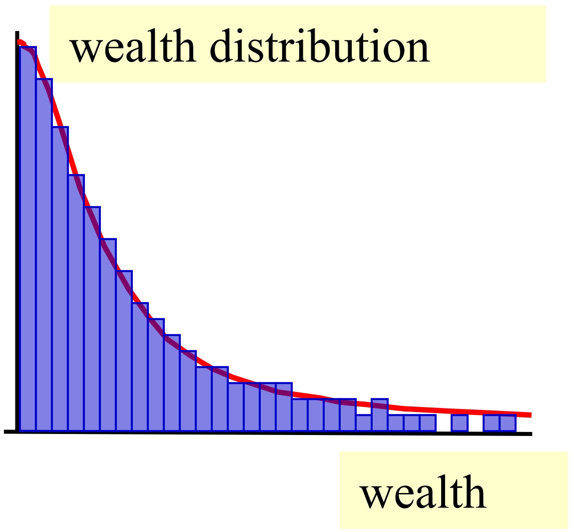
Classical (Boltzmann) gas – Brownian motion;

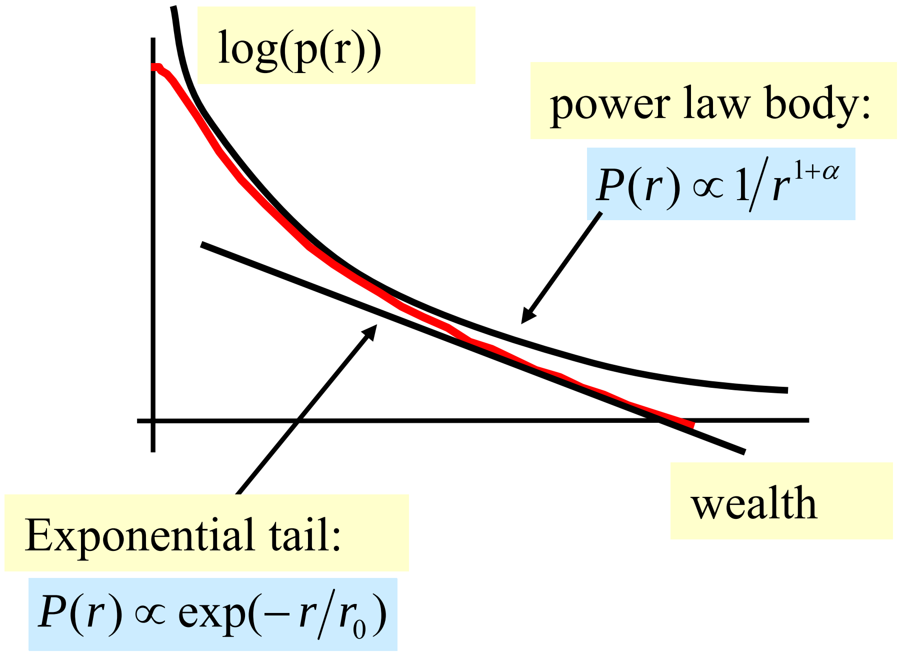
$n(E) \propto \exp(-E/kT)$ $n(v) \propto \exp(-v^2/2kT)$ $P(\Delta X) \propto \exp(-\Delta X^2/\sigma^2)$

Nonclassical (Bose) gas – nonbrownian motion ???;

Pareto law: wealth distribution;

Pareto



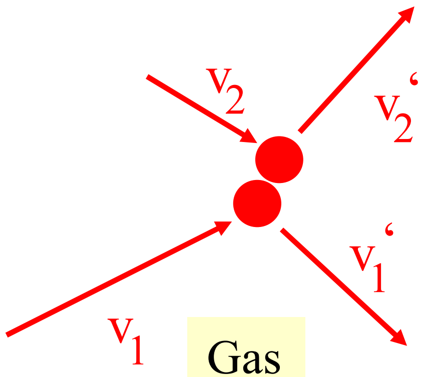


Exponentially truncated Pareto = exponentially truncated Levy

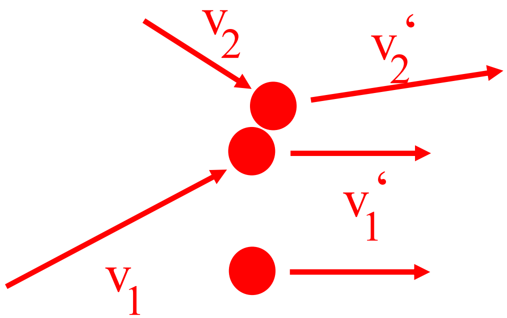
why Pareto distribution ?

Because finance systems are condensates!

Bose-Einstein condensate



Gas



Condensate

Boltzmann:

$n(E) \propto \exp(-E/kT)$

$n(v) \propto \exp(-v^2/2kT)$

$P(\Delta X) \propto \exp(-\Delta X^2/\sigma^2)$

Bose:

$n(E) = 1/\left(\exp\left((E-\mu)/kT\right)-1\right)$

$n(E) \propto \exp(-E/kT)$ for E>>kT

$n(E) \propto (E/kT)^{-1}$ for E<<kT

Pareto:

$P(t) \propto 1/r^{1+\alpha}$ $P(r) \propto \exp(-r/r_0)$

$1 < \alpha < 2$

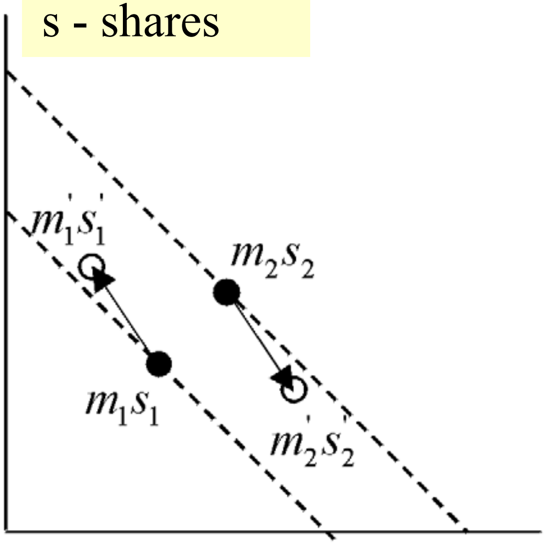
Bose-Einstein:

$n(E) = 1/\left(\exp\left((E-\mu)/kT\right)-1\right)$

$\alpha = 0$

Financial bosons

s - shares



m - money

$$\Delta n_1 = -n_1 \int \left(1+n_1^{'}\right) n_2 \left(1+n_2^{'}\right) d\,(n_1^{'}, n_2, n_2^{'}) + \left(1+n_1\right) \int n_1^{'} n_2^{'} \left(1+n_2\right) d\,(n_1^{'}, n_2, n_2^{'})$$

$$\frac{n_1}{(1+n_1)} \frac{n_2}{(1+n_2)} = \frac{n_1^{'}}{\left(1+n_1^{'}\right)} \frac{n_2^{'}}{\left(1+n_2^{'}\right)}$$

$$n(m,s) = 1/\left(\exp\left(\beta\left(m+s-\mu\right)\right)-1\right)$$

$$E \leftrightarrow (m+s) \qquad \beta \leftrightarrow 1/kT$$

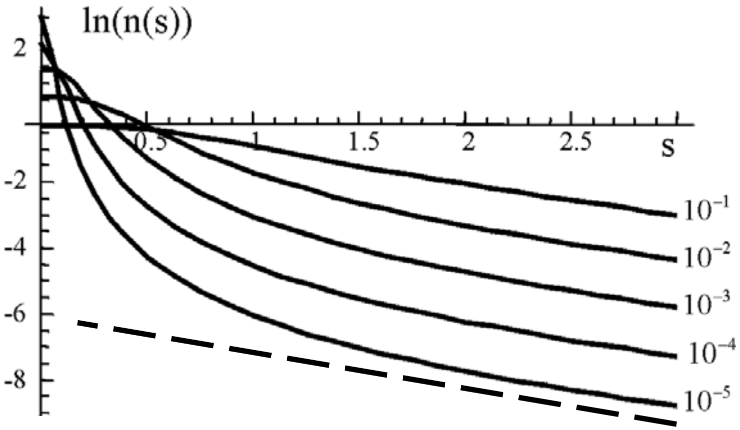
Profit seek: $r^{'}/r = \left(m^{'}+s^{'}\right)/(m+s)$

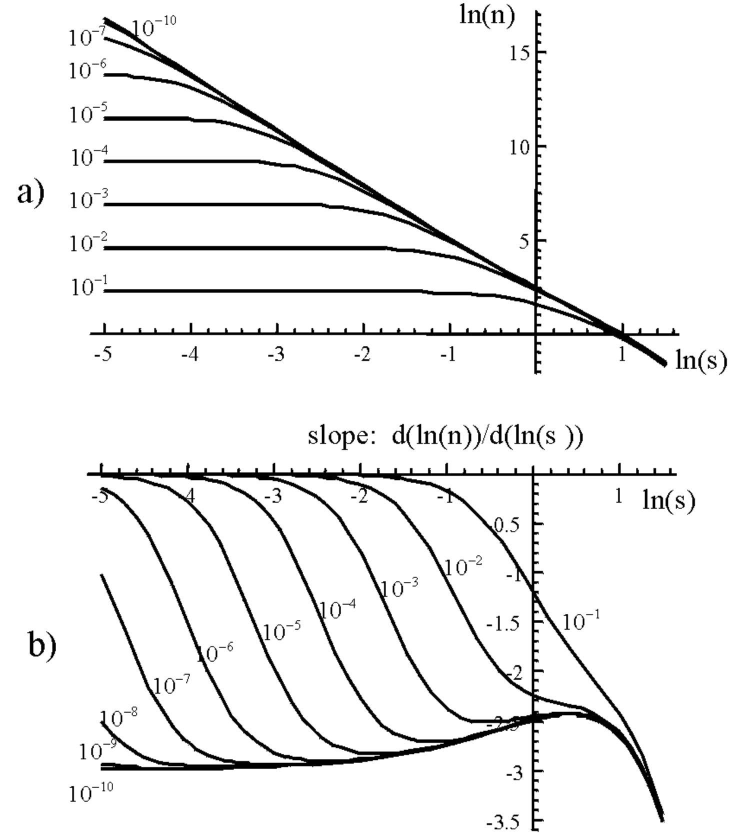
Bosonic enhancement respectively to shares: $\left(1+n(m,s)\right) \rightarrow \left(1+\int n(m,s)dm\right)$

Financial bosons

$$n(m,s)=\frac{(s+m)^2\exp[\beta(\mu_0-m)]}{\exp[\beta(\Delta\mu+s)]-\left(1+\beta s+(\beta s)^2/2\right)}$$

$$n(s)=\frac{1+\beta s+(\beta s)^2/2}{\exp[\beta(\Delta\mu+s)]-\left(1+\beta s+(\beta s)^2/2\right)}$$

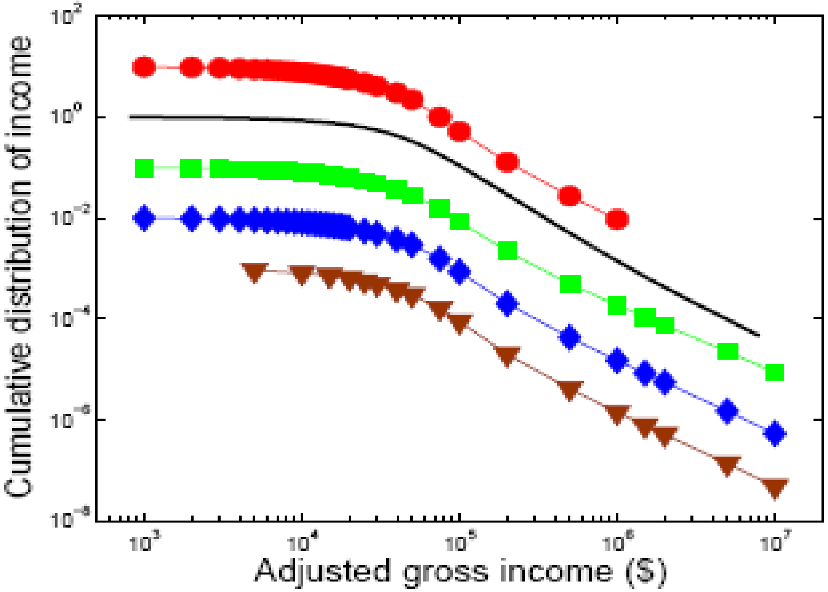




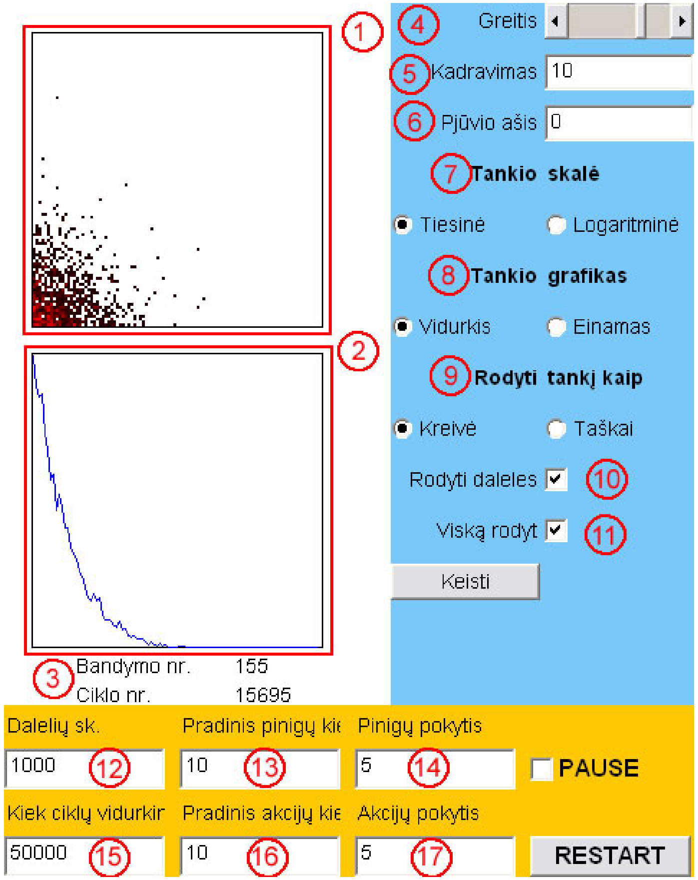
Truncated Pareto!

$1<\alpha<2$

Financial bosons in USA



Financial bosons: Monte-Carlo



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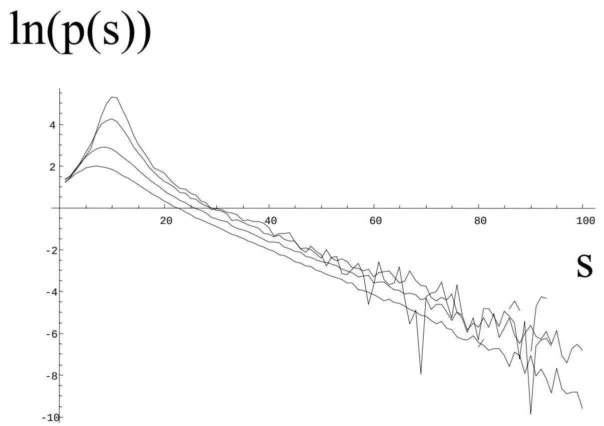

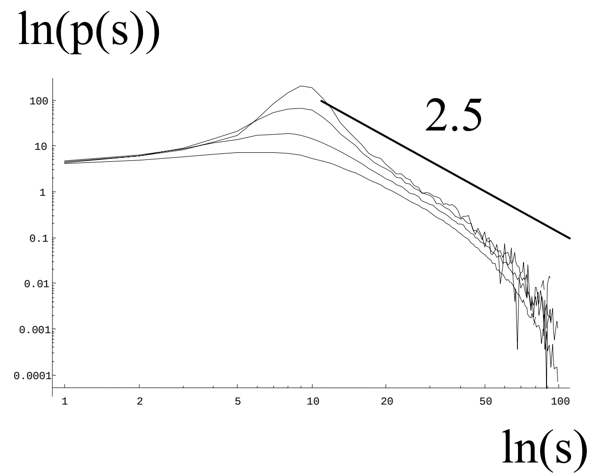

Conclusions:

Bosonic + seek for profit ⟶ Pareto distribution:

$$n(m,s)=\frac{(s+m)^2\exp[\beta(\mu_0-m)]}{\exp[\beta(\Delta\mu+s)]-\left(1+\beta s+(\beta s)^2/2\right)}$$

$\alpha\approx 1.5$

K.Staliunas, Bose-Einstein Condensation in Financial Systems, 2003; cond-mat/0303271

Statistics of price variations? $\alpha\approx 1.5$? Bose diffusion?

- Monte-Carlo simulations (stochastic equation);
- Evolution of distributions (kinetic – master equation);
- Price diffusion vs. diffusion of Bose particle;
- Multicomponent condensate;