

Bose-Einstein Condensation of Financial (Profit Seeking) Bosons

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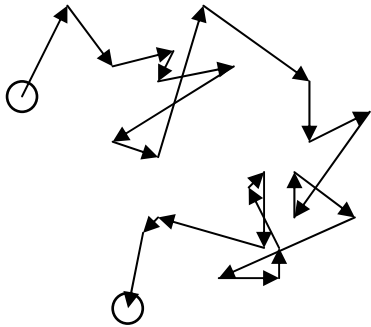
- *Fat Tails (Nongaussian Distributions) in Financial Data;*
- *Fat Tails in distribution of Data-Makers;*
- *Bose-Einstein Condensation;*
- *Thermodynamics of Profit Seeking Bosons;*

Econophysics

- Autocorrelations;
- (Nongaussian) Distributions;
- Crises, financial bubbles;

L. Bachelier 1900.

Price diffusion =
Random Brownian walk!



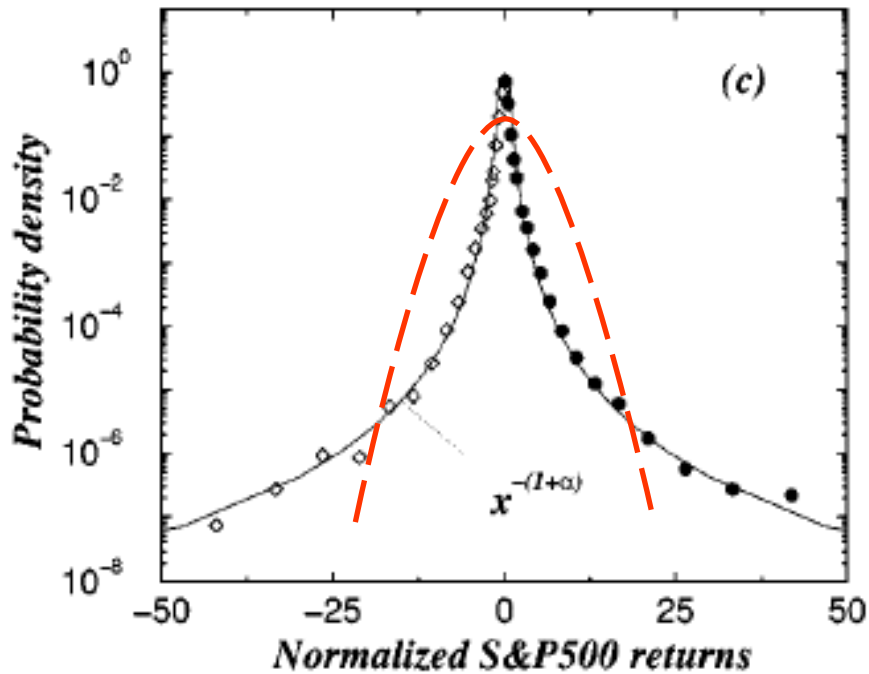
$$P(\Delta X, \Delta t) = \frac{1}{\sqrt{2\pi}\sigma(\Delta t)} \cdot \exp\left(\frac{-\Delta X^2}{2\sigma^2(\Delta t)}\right)$$

$$\sigma(\Delta t) \propto \Delta t^{1/2}$$

Nongaussian

B.Mandelbrot;

~~$$P(\Delta X) = \exp(-\Delta X^2)$$~~



$$P(\Delta X) \propto \frac{1}{|\Delta X|^{\alpha+1}} \quad 1 < \alpha < 2$$

$$\alpha \approx 1.5$$

P.Levy;

$$P(\Delta X) \propto \begin{cases} |\Delta X|^{-(1+\alpha)} & \alpha \approx 3 \text{ ?} \\ \exp(-\Delta X/\sigma) & \text{?} \end{cases} \quad \Delta X \gg \sigma$$

why nongaussian?

Why nongaussian

Nongaussian price changes – nongaussian price changers!

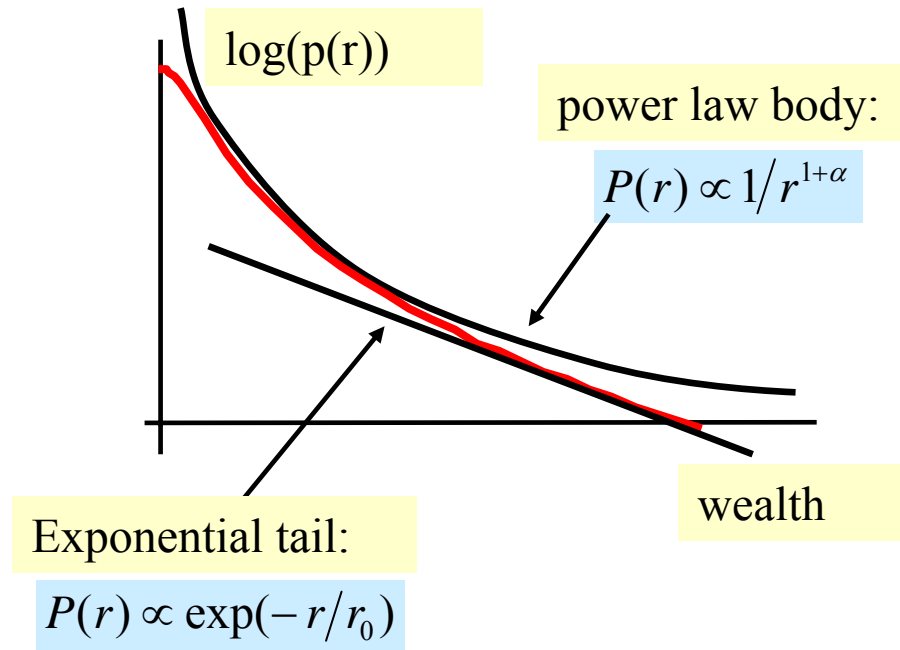
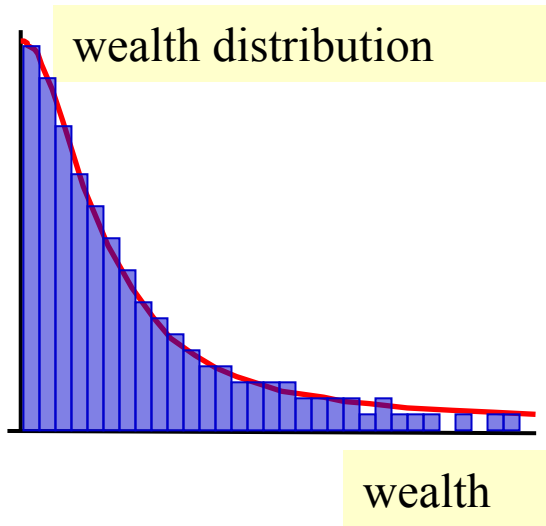
Classical (Boltzmann) gas – Brownian motion;

$$n(E) \propto \exp(-E/kT) \quad n(v) \propto \exp(-v^2/2kT) \quad P(\Delta X) \propto \exp(-\Delta X^2/\sigma^2)$$

Nonclassical (Bose) gas – nonbrownian motion ???;

Pareto law: wealth distribution;

Pareto

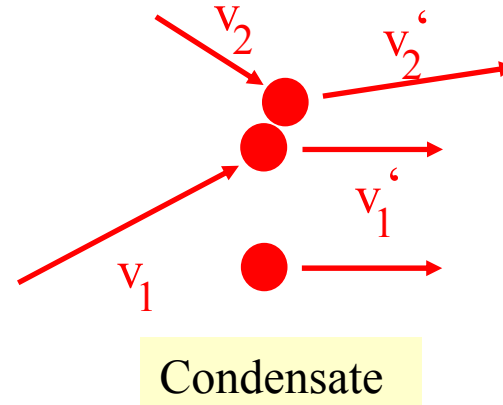
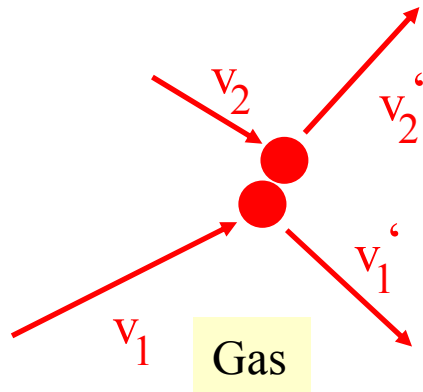


Exponentially truncated Pareto = exponentially truncated Levy

why Pareto distribution ?

Because finance systems are condensates!

Bose-Einstein condensate



Boltzmann:

$$n(E) \propto \exp(-E/kT)$$

$$n(v) \propto \exp(-v^2/2kT)$$

$$P(\Delta X) \propto \exp(-\Delta X^2/\sigma^2)$$

Bose:

$$n(E) = 1/(\exp((E - \mu)/kT) - 1)$$

$$n(E) \propto \exp(-E/kT) \quad \text{for } E \gg kT$$

$$n(E) \propto (E/kT)^{-1} \quad \text{for } E \ll kT$$

Pareto:

$$P(t) \propto 1/r^{1+\alpha} \quad P(r) \propto \exp(-r/r_0)$$

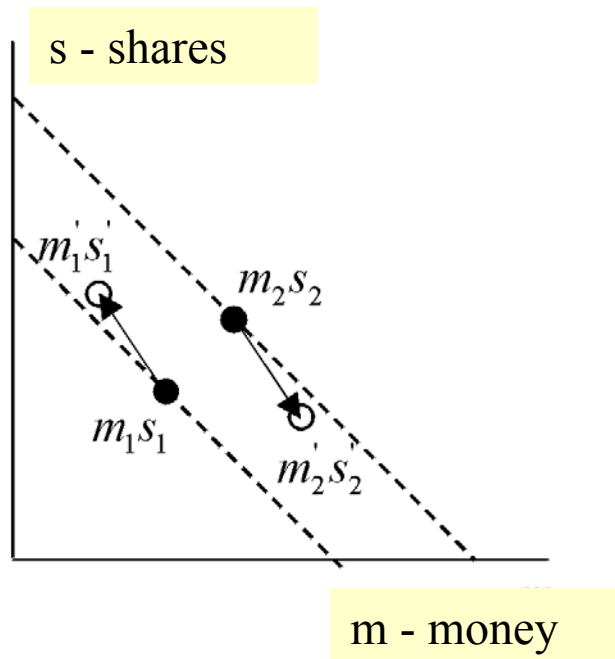
$$1 < \alpha < 2$$

Bose-Einstein:

$$n(E) = 1/(\exp((E - \mu)/kT) - 1)$$

$$\alpha = 0$$

Financial bosons



$$\Delta n_1 = -n_1 \int (1 + n'_1) n_2 (1 + n'_2) d(n'_1, n'_2, n'_2) + (1 + n_1) \int n'_1 n'_2 (1 + n_2) d(n'_1, n'_2, n'_2)$$

$$\frac{n_1}{(1 + n_1)} \frac{n_2}{(1 + n_2)} = \frac{n'_1}{(1 + n'_1)} \frac{n'_2}{(1 + n'_2)}$$

$$n(m, s) = 1 / (\exp(\beta(m + s - \mu)) - 1)$$

$$E \leftrightarrow (m + s) \quad \beta \leftrightarrow 1/kT$$

Profit seek: $r'/r = (m' + s') / (m + s)$

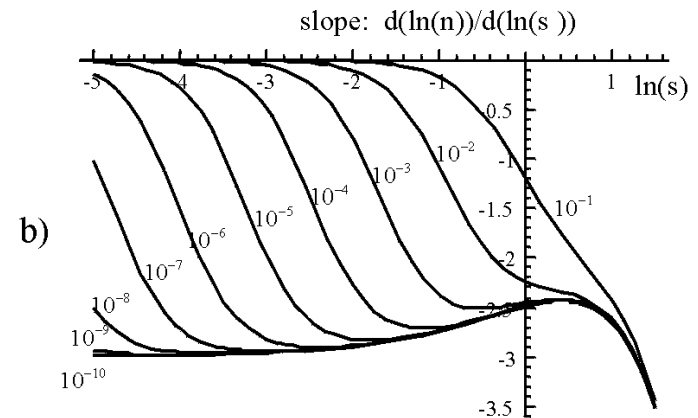
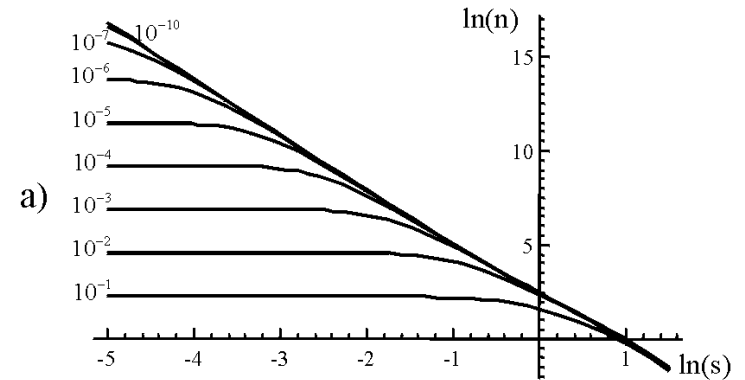
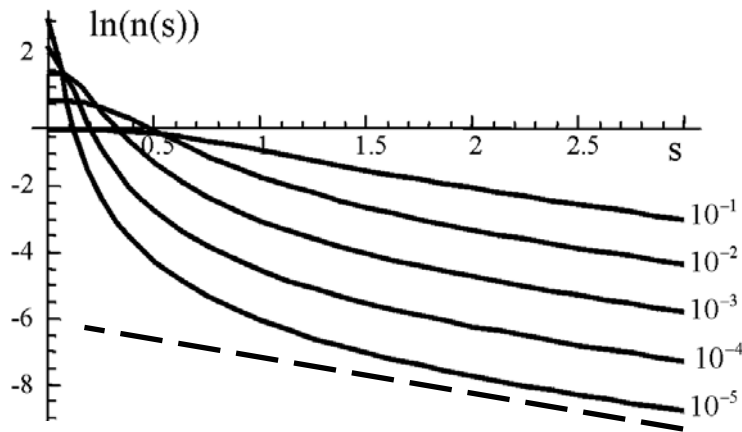
Bosonic enhancement respectively to shares:

$$(1 + n(m, s)) \rightarrow \left(1 + \int n(m, s) dm\right)$$

Financial bosons

$$n(m, s) = \frac{(s + m)^2 \exp[\beta(\mu_0 - m)]}{\exp[\beta(\Delta\mu + s)] - (1 + \beta s + (\beta s)^2 / 2)}$$

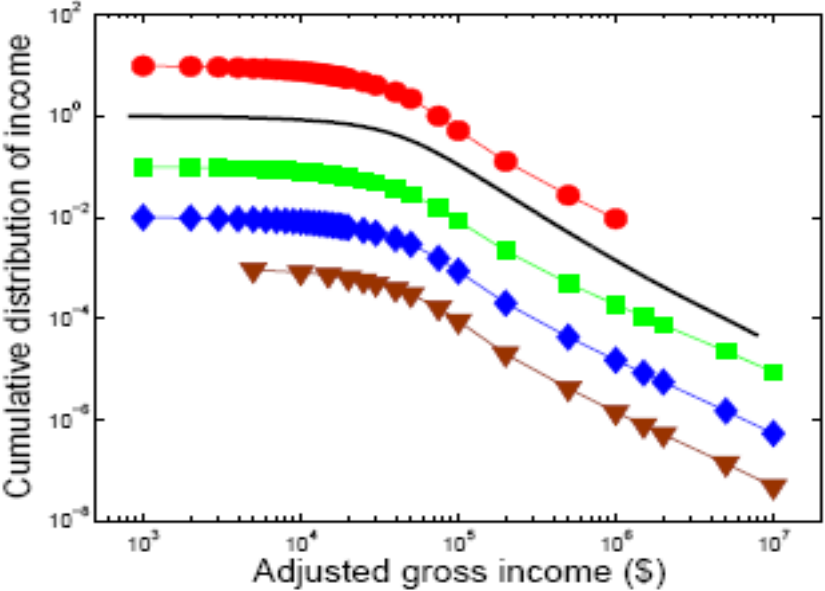
$$n(s) = \frac{1 + \beta s + (\beta s)^2 / 2}{\exp[\beta(\Delta\mu + s)] - (1 + \beta s + (\beta s)^2 / 2)}$$



Truncated Pareto!

$$1 < \alpha < 2$$

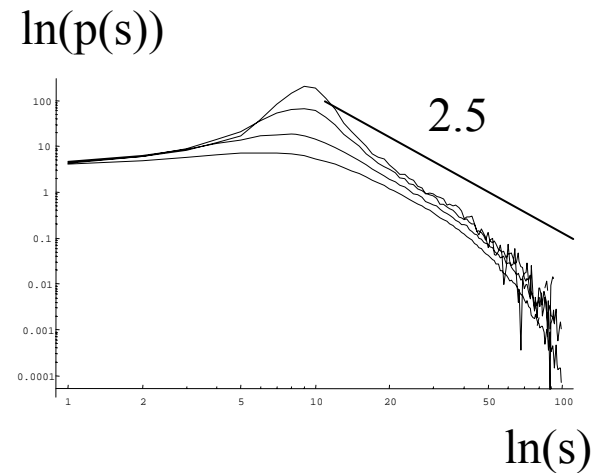
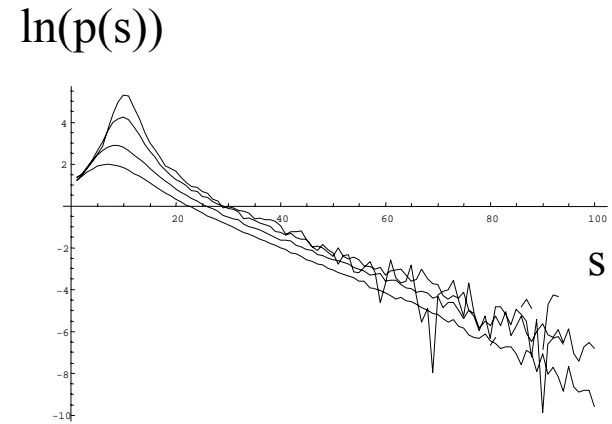
Financial bosons in USA



Financial bosons: Monte-Carlo

The screenshot shows the software interface for the Monte-Carlo simulation. It features a control panel on the right with various settings and two main visualization windows on the left. The top window (1) displays a scatter plot of data points, while the bottom window (2) shows a line graph of the distribution. The control panel includes a speed slider (4), a refresh button (5), a pivot axis input (6), a density scale selector (7) with 'Tiesinė' (Linear) selected, a density plot selector (8) with 'Vidurkis' (Average) selected, a plot style selector (9) with 'Kreivė' (Curve) selected, and checkboxes for 'Rodyti daleles' (10) and 'Viską rodyt' (11). A 'Keisti' (Change) button is located below these options. At the bottom, a yellow control bar contains input fields for 'Dalelių sk.' (12), 'Pradinis pinigų kie.' (13), and 'Pinigų pokytis' (14) with a 'PAUSE' button, and 'Kiek ciklų vidurkis' (15), 'Pradinis akcijų kie.' (16), and 'Akcijų pokytis' (17) with a 'RESTART' button. Test parameters are shown as 'Bandymo nr. 155' (3) and 'Ciklo nr. 15695'.

6.1.1 pav. Programos (iškiepio) aplinkos vaizdas



Conclusions:

Bosonic + seek for profit \longrightarrow Pareto distribution:

$$n(m, s) = \frac{(s + m)^2 \exp[\beta(\mu_0 - m)]}{\exp[\beta(\Delta\mu + s)] - (1 + \beta s + (\beta s)^2 / 2)} \quad \alpha \approx 1.5$$

K.Staliunas, Bose-Einstein Condensation in Financial Systems, 2003; cond-mat/0303271

Statistics of price variations?

$\alpha \approx 1.5$?

Bose diffusion?

- Monte-Carlo simulations (stochastic equation);
- Evolution of distributions (kinetic – master equation);
- Price diffusion vs. diffusion of Bose particle;
- Multicomponent condensate;